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DEMAND FOR FARM TRACTORS IN THE UNITED STATES

A REGRESSION ANALYSIS

Economic Research Service
U.S. DEPARTMENT OF AGRICULTURE

PREFACE

Data related to tractor power are kept up-to-date by the U. S. Department of Agriculture and are published periodically in the Farm Cost Situation. Estimates for 1940-62 are in the November 1964 Outlook issue. Data related to total farm power are also estimated periodically. The writer acknowledges the help of Paul E. Strickler in obtaining current estimates.

Washington, D.C. November 1966

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SUMMARY

Total farm power available to farmers increased from about 40 million horsepower in 1920 to 385 million in 1962. Tractor horsepower (maximum belt) increased from 500,000 to 162 million during this same period. The demand for additional new tractors evolved from many individual farmers maintaining or adding to their stock of tractors. In this report, a single-equation regression model is used to explain aggregate tractor horsepower purchases for the United States between 1920 and 1962. Tractor horsepower purchases are expressed as a function of economic, technological, and personal preference variables. They include tractor horsepower on hand, crop production, ratio of tractor prices to prices received for products sold, size of new tractors purchased, age of tractors, and number of farms.

Both short- and long-run elasticities of all the independent variables are estimated. Long-run elasticities are estimated by considering the influence of earlier time periods. The elasticity of demand for tractor horsepower purchases with respect to the real price of tractors (ratio of tractor prices to prices received for farm products sold) ranges between 1.7 and 2.7. The differences between the short-run and long-run elasticities suggest that 89 to 93 percent of the adjustment is completed in the first year. This elasticity of demand with respect to the real price of tractors has been lower in recent years.

The average relationships between tractor purchases and economic and technological variables are used to project tractor purchases into the future. Projected purchases for 1970 are about 8 million horsepower (maximum belt horsepower), up from about 7½ million in 1962, if the following conditions prevail: (1) Crop production increases about 15 percent(index of crop production, 1957-59=100, up from 108 to about 124); (2) ratio of tractor prices to prices received for farm products sold increases about 7 percent (index of tractor prices divided by the index of prices received, 1957-59=100, up from 110 to 118); (3) size of tractors purchased increases from 54.5 to 80.0 horsepower; and (4) number of farms decreases from 3.7 million to 3.3 million. The number of tractors sold in 1970 will be about 100,000 units, a decrease from the 150,000 units sold in 1962. A wide variation in projected purchases results from considering relatively small differences in the independent variables. However, changes in these variables tend to offset each other.

This projection in tractor horsepower purchases assumes a reduction in the number of tractors on farms. Tractor numbers would decrease from 4.7 million tractors in 1962 to 3.7 million in 1970.

The large weight assigned to farm numbers weakens this model for making projections. This projection assumes that changes in technology (primarily mechanization) associated with the reduction in the number of farms will remain the same in the coming years. It is more likely that changes in technology will become less important. Therefore, a continuing reduction in the number of farms will underestimate horsepower purchases.

Farm power will continue to be an important factor in agricultural production. It is indeed doubtful that changes in the next 40 years will be of the same magnitude as those in the preceding 40 years. In future years, it is likely that total farm power available for use will increase at a lower rate. Although tractor power increased throughout the period of study from about an eighth of the total power in 1920 to 40 percent in recent years, it is expected to level off while other sources of power will continue to increase. Thus, tractor power will be a smaller proportion of total farm power in the future than it has been in the recent past.

DEMAND FOR FARM TRACTORS IN THE UNITED STATES--A REGRESSION ANALYSIS

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INTRODUCTION

Power has always been an important factor in agricultural production. The kind of power used and the contribution of each kind of power have changed over time. In this century, the sharp shift from use of manpower and horsepower to mechanical and electrical power has added much to total farm power.1/ This report presents a brief description of total farm power in the United States during 1920-62, and analyzes in depth the aggregate demand for farm tractor power during the period. Regression analysis is used to estimate the relative importance of economic and other related factors to tractor horsepower purchases and these relationships are used to project purchases by farmers for 1970. The observations made in this report will add to our knowledge of the probable future demand for farm tractor power.

The appendices include a discussion of other approaches considered before settling on regression analysis. A recursive linear programming framework was considered, and Markov processes that did not include a system of rewards were used to project tractor numbers.

Total Farm Power

In the early 1920's, about 40 million horsepower was available for use on farms and more than half of it derived from horses and mules (fig.1). At that time, tractors provided only about 5 million horsepower. Smaller quantities, between 1½ and 3 million horsepower, were available from each of the following: farmworkers , steam and gasoline engines, farm trucks, and electric plants on farms (16).2/ Windmills on farms and mounted engines on harvesting machines provided additional sources of farm power.

By 1962, power available for farm use had increased to about 385 million horsepower, or almost 10 times greater than in 1920. Tractors and trucks accounted for most of this increase. Horsepower available from farm tractors and trucks was 162 and 144 million horsepower, respectively, or an average of nearly 40 times as great as in the early 1920's.

1/ In this report, farm power includes man, animal, mechanical, and electrical power, and the force or energy available to do farmwork.
2/ Underscored numbers in parentheses refer to items in Literature Cited, p. 48.

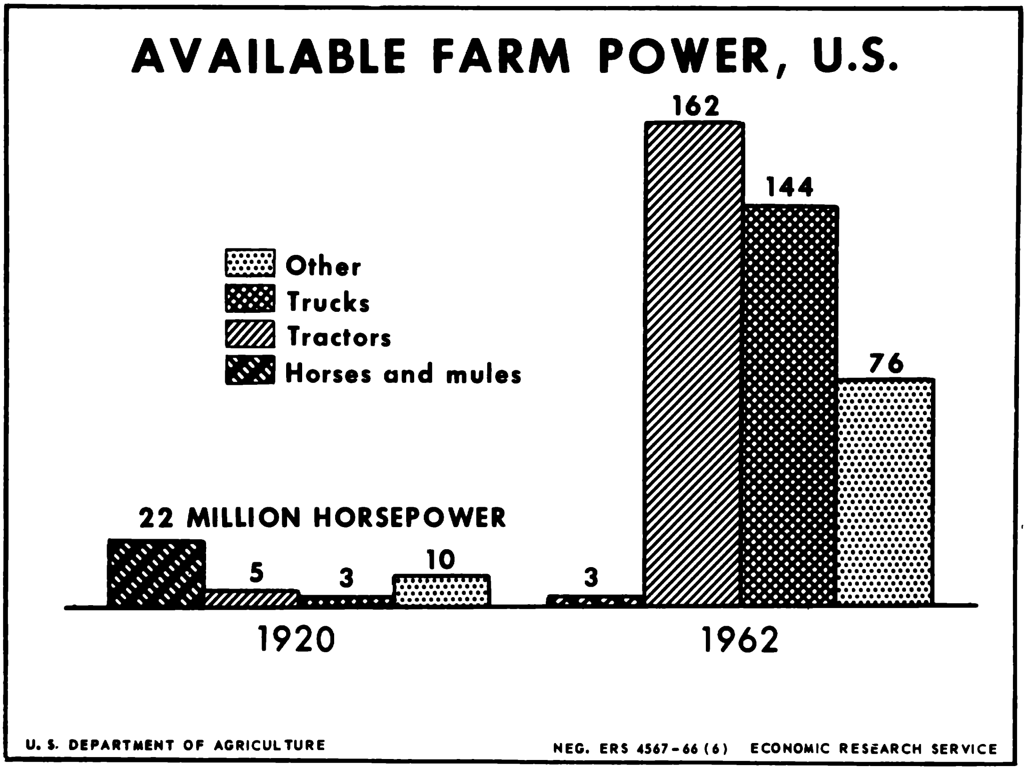

Figure 1

Throughout the 42-year period, mechanical power was available from mounted engines on some of the harvesting machines as well as from tractors. Initially, the mounted engines were mostly used on combines (harvester-threshers), and later on balers and forage harvesters. They are still being used on all these harvesting machines but to a lesser extent. With the advent of the larger tractors, more of the newer machines are now operated by power takeoff . Also, some of these machines are operated with hydraulic motors in which the source of power is the tractor. Thus, formany farm machines the trend is away from mounted gasoline engines. Mounted engines provided about 120,000 horsepower as early as 1920, and an estimated 20 to 25 million horsepower in recent years.

Self-propelled machines which came into general use after World War II have provided increasingly greater power. By 1958, about half of the 56,000 combines sold were self-propelled (25). These machines added about 23,000 self-propelled units to a stock of 208,000 similar units. Self-propelled cornpickers and forage harvesters were much less frequently sold; in 1962, less than 20,000 of these units were on farms. It is estimated that in recent years about 15 to 20 million horsepower have become available to farmers from self-propelled equipment.

Electricity has provided farmers with an additional source of power. It was first used primarily for lighting but came into more general use in the 1940's for jobs such as grinding feed, cleaning gutters in dairy barns, unloading silage, drying grains and roughage, and feeding livestock. Electrical power consumed on farms to do mechanical work, compared with that used to heat water or provide light, has been rising rapidly. Electric motors have recently made roughly 7 million horsepower available to farmers for mechanical work, in addition to the 3 million horsepower equivalent used for nonmechanical work.

The general increases in power available for farm use occurred while manpower, as well as horse and mule power, was being replaced by mechanical and electrical power. As a result of this change, fewer farmworkers produced more agricultural products with less hours of labor per worker. From 1920 to 1962, the number of farmworkers was reduced by about half--from 13.4 million to 6.7 million (30)--and the man-hours of labor were down from about 24 billion hours to 9.1 billion (29). Most of the decrease in man-hours of labor occurred after 1944, but the decline in the number of farmworkers was about the same throughout the entire period. Total hours per person employed 3/ declined from about 1,800 to 1,400 during the 42-year period; in the early years they remained about the same but declined rapidly after the depression years 1931-34. Numbers of horses and mules decreased from an all-time high of about 26 million in 1920 to less than 3 million in 1962 (6, 16). The number of horses used decreased throughout the period, and now most of the horses still on farms are used less than they were earlier.

Tractor Power on Farms

Tractor power on farms has been increasing since tractors became an integral unit in many farm operations (fig.2). Until after World War II, the increase in horsepower was largely a result of more tractors on farms. Much of the added horsepower in postwar years came from larger tractors. In more recent years, nearly all of the increases in horsepower have come from larger units; evidence is that the number of tractors being scrapped now is about equal to the number purchased.

In 1912, all the tractor power used on farms was from about 8,000 tractors. Fifty years later, in 1962, tractor power was supplied by almost 4.7 million tractors. Maximum belt horsepower (not including garden tractors) available to farmers increased from about 400,000 in 1912 to 162 million in 1962.

Tractors on farms in the early years studied were used for heavy field operations such as plowing and disking, and for providing stationary power for grain threshers. Later, tractor power replaced horses for operations such as harrowing and pulling harvesting machines through the fields. After World War II, tractors replaced nearly all of the horses formerly used for light work such as planting, cultivating, mowing, and hauling crops from the fields to the farmstead.

The kinds and quantities of tractor power used in different regions of the country vary with the combination of labor, capital, and available natural resources such as moisture, temperature, topography of land, and type of soil. The first concentration of tractors appeared in the Middle West, an area where natural conditions and types of farming favored the use of mechanical power. Tractors became more commonplace in all parts of the country with the development of row-crop tractors which could be used for almost all farm operations. They were purchased sooner in areas that had less available labor, such as in the Northeast and North Central regions, where competition of industrial users for farm labor was more apparent than in Southern regions.

3/ Persons employed are an annual average of those working during the last calendar week each month ending at least 1 day before the end of the month.

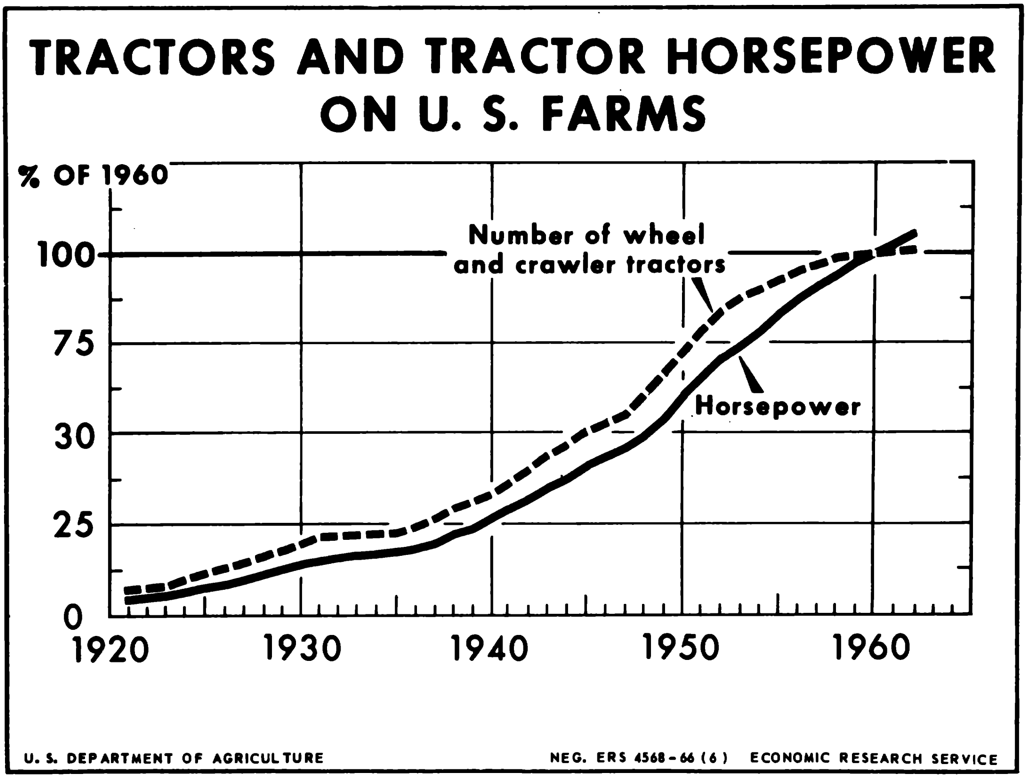

Figure 2

Until recently, the distribution of number of tractors gave a good description of total tractor power throughout the country. This picture is being clouded by the sales of many larger tractors to specialized types of farmers in different farming areas. In general, more of the larger and fewer of the smaller tractors are bought in the North. The opposite is true in the South, and the West is in between.

The distribution of tractor power throughout the United States varies with the proportion of cropland harvested. Concentration of tractor numbers is most dense in the Corn Belt, where about half of the total land area is harvested (fig.3). In the Mountain States, where only about 5 percent of the total land area is harvested, the concentration is light.

THE TRACTOR MARKET

The aggregate sales and exchange of new and used tractors are all part of the tractor market. Farmers and users of industrial tractors adjust their stock of tractors continually. The many individual farms maintaining or adding to their stock of tractors cause a flow of both new and used tractors. Although tractor stocks remain fairly constant from year to year, incentives which encourage small changes in the stock result in large changes in flows or purchases. The net flows of wheel and crawler tractors in any one year are in reality the demand for tractor power.

New tractors move from the manufacturer through the dealer to farms or industrial businesses (10). Used tractors move between farms and industrial businesses directly through farm sales and individual transactions, and indirectly through machinery dealers.

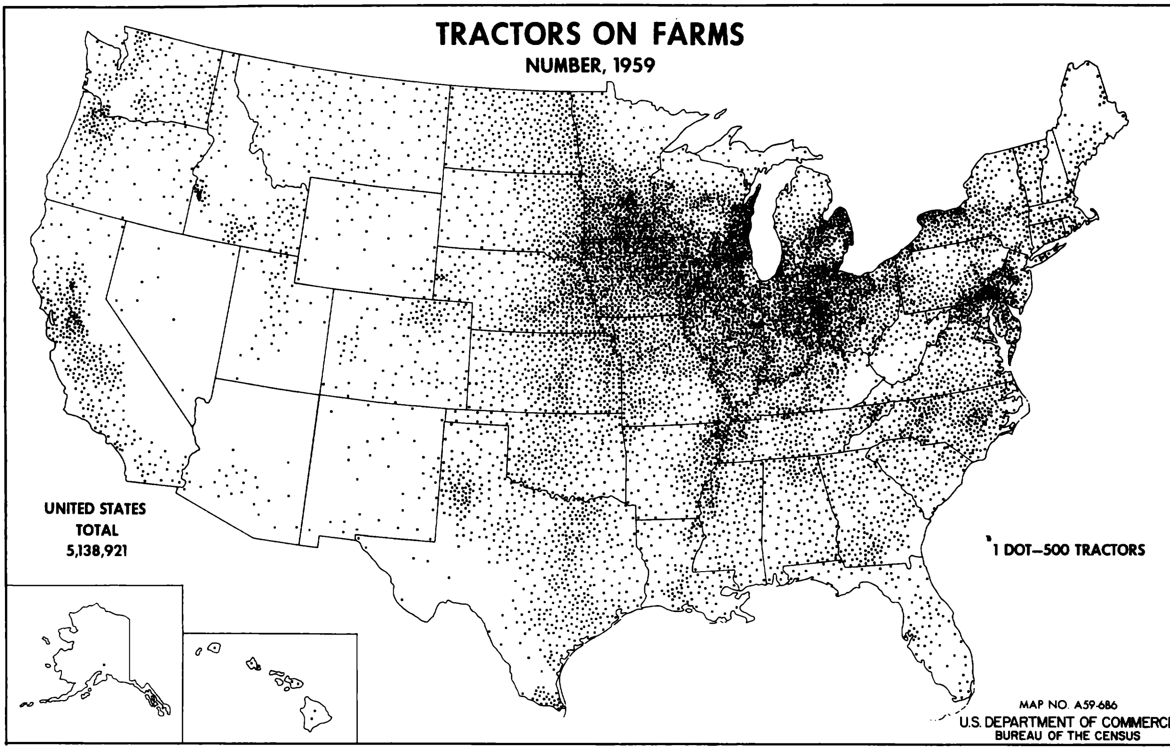

Figure 3

Flow of New Tractors

In the early 1920's, nearly all of the tractors used domestically were wheel tractors and were sold for farm use. In 1962 nearly 10 percent of all tractors sold domestically were crawler tractors and 20 percent were sold for nonfarm use. The farm market traditionally has been the primary outlet for wheel tractors and in 1962 still accounted for about 85 percent of all wheel tractors sold (fig.4).

Although the market for farm tractors has passed its peak, the market for industrial tractors continues to expand. Tractor sales for industrial purposes doubled from about 15,000 tractors in 1940 to 30,000 in the early 1960's. They are still increasing. As the market for industrial tractors expands and that for farm tractors remains the same or becomes smaller, tractors may be designed specifically for industrial jobs and may then be adapted to farm use. In the past, all tractors except the largest industrial units were designed for farm use and then adapted for industrial uses.

New tractors for both farm and industrial uses are available from manufacturers within and outside the United States. Except for 3 years in the midthirties, when an average of about 2,000 tractors were imported (32), imports of farm tractors were unimportant until after World War II. After the war, imports climbed rapidly and then leveled off. Imports of tractors for farm use averaged about 9,500 units from 1947 to 1962, and represented about 6 percent of sales to farmers in the early 1960's. Nearly all of the imports (about 97 percent) of farm tractors were from Canada and the United Kingdom. The imports were primarily wheel tractors and increased in horsepower much the same as domestic production. In 1962, about 40 percent of the imports were 50 horsepower or more.

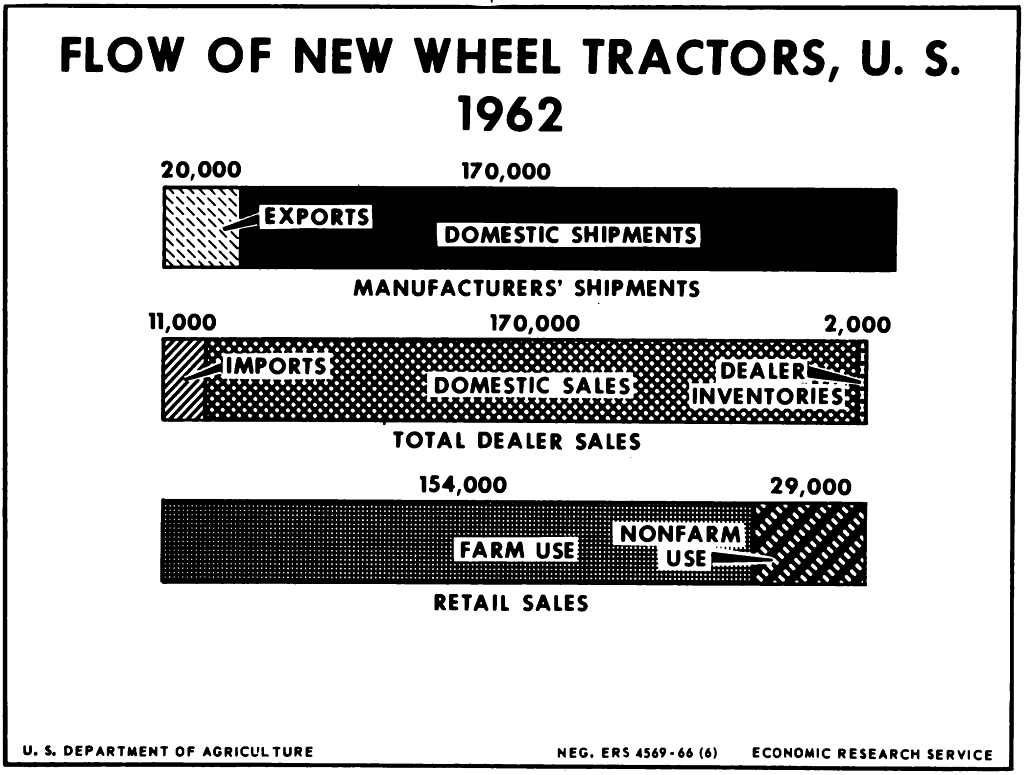

Figure 4

The market for tractors by U.S. manufacturers includes export sales in addition to local sales to the farm and industrial market. The number of tractors exported was about 25 percent of total U.S. production during the 1920's. The export market has been and is continuing to be an important part of domestic production. From 1937 to 1957, exports ranged from 13 to 22 percent of U.S. production and averaged about 17 percent per year (31). In the most recent years, actual exports and exports as a percentage of U.S. production have been lower.

Most U.S. exports of new tractors in recent years have gone to Canada. The Canadians are buying about one-fourth of all crawler tractors and three-fourths of all wheel tractors exported. The size of tractors exported has increased much the same as those used domestically. In 1962, about 35 percent of the crawler tractors exported were 100 horsepower or more and about 70 percent of the wheel tractors were 50 horsepower or more.

Recently, new tractors produced for both farm and nonfarm use have been much larger. The average horsepower rating of new tractors purchased by farmers increased from about 20 horsepower in 1920 to 27 horsepower (maximum belt) in 1940.4/ In the latter year, about 85 percent of the new tractors were less than 30 horsepower. The size of new tractors increased rapidly since then. In 1962, three-fourths of the new tractors were 45 horsepower or more and only about 15 percent were less than 35 horsepower. The average horsepower rating was about 50 horsepower.

4/ Strickler, Paul E. Unpublished estimates based on shipments weighted by size groups.

The farm market over time includes a changing number of farms of widely differing characteristics in any one time period. This does not preclude common characteristics among many individual farms as well as among groups of farms. Farms are often grouped as to level of income and type of farm. These groups have widely different purchasing habits when buying tractors.

New tractors are more frequently bought by farmers with higher gross incomes (27). In general, these farmers buy much more than their proportionate share of large tractors and about their proportionate share of small tractors.5/ On the other hand, farmers with lower gross incomes buy much less than their proportionate share of large tractors and about their share of small tractors. Cash grain and vegetable farmers are the primary purchasers of new tractors.

Market Flow of Used Tractors

The market flow of used tractors is affected by changes in the flow of new tractors and the number of tractors scrapped. It includes flows to both the farm and industrial market. The flow of used tractors between these two markets has been relatively small thus far, but it is likely to increase in the coming years with the growing importance of tractors in the industrial market. Used tractors flow between individuals directly through farm sales and individual transactions, and indirectly through dealers. The used tractors moving through dealers are primarily limited to trade-ins for other new and used tractors or other farm machines. Few used tractors are purchased outright by dealers, and many of the older tractors that are traded in are scrapped. Some farmers and industrial users of old tractors scrap them directly instead of trading or selling them to dealers.

Used tractors tend to move between firms within an area; there is little movement between areas. Similarly, there are few direct sales or exchanges of used tractors between the United States and other countries. Used tractors began moving out of this country during the early years of World War II. They reached a level of about 2,000 units by the end of the war and have remained near this level. Most of the used wheel tractors have been shipped to Canada and Mexico, with Mexico getting about twice as many as Canada.

EARLIER STUDIES

Several published studies describe the demand for farm machinery by use of econometric models (5, 13, 14). All of these deal in part with or are related to the demand for farm tractors.

In these studies, the demand for farm tractors is a quantity concept. Capital expenditures for farm tractors are deflated by an index of price change. Thus, the quantity of tractors purchased (flow of tractors in investment theory) is the constant dollar value of purchases. Some discussion is directed toward the estimates of quantities of tractors purchased. The difficulties of using presently calculated price indexes

5/ The proportionate share assumes that the proportion of tractors of a specified size to all tractors and of specified farms to all farms are the same.

for deflating expenditures are explored. But for lack of alternatives, a form of price index is finally used to deflate capital expenditures to get estimates of quantities purchased.

Economic variables were selected in these studies in an attempt to measure changes in the real price paid for tractors, changes in costs of factors used in production, and changes in assets and liabilities. Heady and Tweeten (14) include time as an additional variable in most of their models.

In these studies attention is directed to adjustments in the changes in demand. They consider either the adjustments made in the first year only and compare it with the total adjustment, or the adjustment made in only one variable for several years. These adjustments explain the differences between long- and short-run estimates of elasticity of demand. 6/

Griliches (13) also investigates the demand for the stock of tractors on farms. 7/ On the other hand, Cromarty (5) and Heady and Tweeten (14) compare results derived from a single-equation model with those derived from a system of equations.

SCOPE OF THIS STUDY

This report has been restricted to tractor horsepower on farms because of the limited data available for related farm inputs. 8/ It includes information related to the purchase of a durable input from the time of the technological innovation to the present, except for the war years. The study was limited to years after 1920 because of the very small number of tractors on farms before World War I. The model used here is applied only to U.S. data. 9/

6/ The adjustment coefficient shows the percentage of the total adjustment that will be made in the short run. The short-run estimate of demand is the change in purchases made now because of changes in a specified independent variable. The long-run estimate is the change in purchases that would be made over a period of years because of changes in the independent variable. This change in purchases over the years assumes that there have been no further changes in any of the independent variables.

7/ In this formulation, purchases of tractors are important only in the way they affect the number of tractors on farms. The annual input of tractors on farms reflects the quantity on hand rather than the quantity purchased. In investment theory, this is called the stock and flow problem; the stock of tractors is the quantity of tractors used in farm production, and the flow is the number of purchases minus the replacement.

8/ In studying the demand for a durable input, it is common to think in terms of a broad grouping of inputs because if the input, such as available tractor power, is narrowly defined, it may be more difficult to measure the influence of the many closely related variables. An alternative study might include all power inputs, such as man-labor and horsepower, power from engine-mounted and self-propelled machines, and power from electric motors. A common measuring unit might be horsepower equivalents of man-labor, horse labor, mechanical power, and electric power. In terms of farm inputs, the ideal measure would include horsepower equivalents available and intensity of use.

9/ An alternate and more comprehensive procedure would be to work with regional data by economic class of farm and then sum the regions to get an overall estimate. The regional data by economic class of farm needed for using this approach simply are not available, and could not feasibly be derived for any earlier period of years. The possibility of using a cross sectional analysis for a single period in time has some merit, particularly where shortages of historical data are evident.

Values for each variable, along with the definition and their sources, are presented in appendix I. They include annual estimates for 1921-41 and 1947-62. The war years are omitted because of the limited purchases and regulated prices. Farmers' actions during the war were not influential in determining the quantity or prices of tractors being sold.

In this study, the years 1942-46 are considered the World War II years. The year 1941 was included in the data because American involvement in the war did not begin until the last month and production control of farm machinery did not begin until later. Purchases of tractors were not restricted and prices for products either bought or sold were not regulated. The year 1946 was not included in the study because production of farm tractors in that year was limited due to the shortage of supplies and the time required to change production plants to peacetime activities. Further, an unusually high number of farm tractors were old and farmers were willing to buy more for a given situation than they had been in earlier years or would be willing to buy in later years. Annual data are used for the units of observation. 10/ The calendar year is used (1) because it seems logical in the decision-making process for consumer durable goods, and (2) because historical data used in this analysis are available only for calendar years.

METHOD OF ANALYSIS

A single-equation regression approach is used to describe the demand for farm tractor power.11/ The procedure is first to describe the economic, technological, and personal preference factors that affect demand for farm tractors and then to formulate models that identify relationships within the market. Solutions from these models provide statistical estimates that describe the average relationships when considering individual years, and also when considering one or two earlier years. 12/

10/ A note on the units of observation for a durable good appears in appendix II.

11/ A complete system of equations could also be used to describe demand for all mechanical power available to farmers if information on all sources of power available to farmers were complete. A comprehensive discussion of complete systems is presented by Foote (12). With this procedure, each source of mechanical power could be included in the system as a separate variable. Thus, the power sources would be the endogenous variables, and their values would be determined simultaneously while considering the influence of variables outside the system and lagged variables inside the system. As this problem is stated, variables outside the system would be the same ones used in a single-equation model. This system of equations could also be expanded to have additional variables in the system. That is, some variables that were considered predetermined might be mutually determined. This is particularly true of the price variable. It could be argued that the price of tractors also depends upon quantities purchased instead of purchases depending solely on predetermined prices as assumed here.

12/ An elaboration of several other approaches involving recursive linear programming techniques and Markov processes that were considered here is included in appendix III.

This report differs from earlier studies of demand for tractors in three ways: (1) Demand for tractor horsepower is estimated directly instead of by deflating annual expenditures or dollar inputs. The tractor horsepower purchased for use on farms is the number of tractors times their respective sizes. This concept of quantity eliminates the necessity for deflating the data; therefore, it is not affected by selection of a deflator. (2) In addition to variables reflecting changing economic conditions, others are included to reflect changes in farmers' technology and in tastes and preferences. Among these variables are size of tractors purchased, age of tractors on farms, and the number of farms.13/ (3) Long-run estimates of the demand elasticities are calculated by analyzing lagged values for all the variables over several years. The adjustment path of the changes in demand elasticity over time is a by-product of this procedure. No assumption is made about the form of the distributed lags. That is, successive regression coefficients for any or all variables over time are estimated by least-squares procedures, without specifying any relationship between them. A good explanation of this and other approaches used to analyze distributed lags appears in Nerlove (22).

Factors Associated With Tractor Horsepower Purchases

This analysis suggests that purchases of farm tractor power depend upon some price and income variables, depreciation, technology, personal preferences, and number of farms. These factors are somewhat interrelated and difficult to separate. Their influence may be more easily evaluated by expressing purchases as a function of horsepower on farms, crop production, price and income variables, size of tractors purchased, age of tractors on farms, and number of farms (fig.5). The ratio of tractor prices to prices received for farm products sold gives an indication of changes in relative prices and at the same time is a measure of income. The horsepower of tractors on farms and crop production give clues to the amount of depreciation and changes in technology and personal preferences. The number of farms, size of tractors purchased, and age of tractors on farms also reflect changes in technology and personal preferences.

Two primary goals were considered in selecting factors associated with changes in purchases of tractor horsepower. The first was to include variables which were thought to have a causal relationship with horsepower purchased and to be able to measure the importance or contribution of each of these. The second was to include variables which would facilitate using the model for making longer term projections.

Because of the second goal, two variables, like crop production and prices, were included rather than a single variable such as income. When using a regression model for projecting into the future, individual projections of each of the independent variables must be made to get the projected value of the dependent variable.

13/ It is recognized that the size of tractors purchased is used in estimating the dependent variable and is then included as an independent variable. It was hypothesized that the effect was not important in this problem since the size of tractors purchased and tractor horsepower on farms did not appear to be closely related. This later proved to be correct. The relationship between horsepower purchases and size of tractors purchased is discussed in the first part of the section dealing with results.

ASSOCIATION OF FACTORS RELATED TO PURCHASES OF FARM TRACTORS

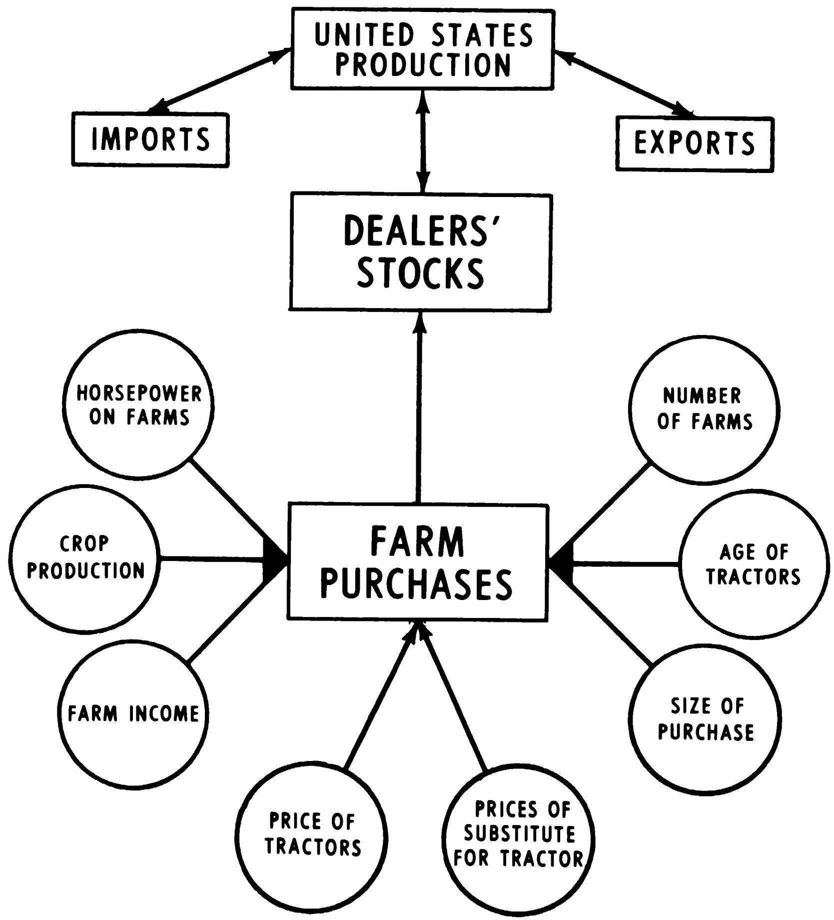

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Figure 5

With the data analyzed here, individual projections of crop production and prices in the earlier years contributed to projections of tractor horsepower purchases that more nearly approached actual purchases than when projections of income were made. Past relationships show that deviations from the general trend lines are much greater for income than for crop production or the price ratio used here.

Further, the projections in this report are average estimates for 5 or 10 years from now, not estimates for individual years. Again, one may observe that crop production and the price ratio discussed here have been generally increasing in the longer term periods and that they readily lend themselves to projections. This is not the case for income which has generally fluctuated, with not much of an indication of where it is going.

Variables were also selected in formulating the model to help explain differences in purchasing patterns over time. For example, number of tractors already on farms and size of purchased tractors were included as variables because they will certainly have a bearing on number of tractors and total horsepower purchased in the future. During a period when the number of tractors on farms is increasing rapidly, one would expect continually higher purchases. When the number of tractors is leveling off or even declining, replacement sales will continue high; larger but fewer tractors will replace their obsolete and worn-out counterparts.

The number of farms and the size of tractors purchased were included as variables because the continuing reduction in number of farms is closely related to the exodus of farm labor and encourages the use of larger tractors and more total horsepower per farm. The continuing trend toward fewer farms may well result in fewer tractors and even less horsepower per 100 acres of cropland harvested. In total, then, these variables will tend to show likely changes in purchasing patterns which will result from changes in both the physical and economic environment.

The arrows in figure 5 on page 11 show the direction of influence of the factors affecting the aggregate demand for farm tractor power. They suggest that farm purchases are independent of dealers' stocks except when there is a change in the price of tractors.

Dealers' stocks of tractors are influenced by farm purchases, by domestic production of tractors, and by imports. However, these stocks also influence the imports and number of tractors produced. Since all tractors are sold through dealers, any disparity between production and farm purchases will show up in dealer inventories. These inventories have fluctuated considerably in recent years. New tractors in dealer inventories reached an alltime high of about 179,000 tractors in 1959. Since then, they have declined and are now around 100,000.

In the long run, tractor production is primarily influenced by the dealer's stock of tractors. Manufacturers regulate production to attempt to maintain a constant supply of tractors.14/

The production schedule is geared indirectly to the overall economy and directly to the farm economy. Output is based on physical and economic conditions in the preceding year and on expectations for the present and future years. Thus, output in any one year is largely predetermined, and changes in prices are associated with changes in costs.

Regression Model

A single-equation regression model is used to describe the economic relations between tractor horsepower purchases and factors associated with purchases.15/ The functional relationship is assumed to be:16/

$$(1)\ H_{(t)}=a_0 + a_1H_{(t-1)} + a_2C_{(t)} + a_3R_{(t)} + a_4S_{(t)} + a_5A_{(t)} + a_6N_{(t)} + u_{(t)}$$

where

$H_{(t)}$ = horsepower purchases this year.

$H_{(t-1)}$ = horsepower on farms in the beginning of this year.

$C_{(t)}$ = crop production this year.

$R_{(t)}$ = index of tractor prices divided by prices received this year.

$S_{(t)}$ = average size of tractors purchased this year.

$A_{(t)}$ = average age of tractors on farms this year.

$N_{(t)}$ = number of farms this year.

$U_{(t)}$ = residual term.

14/ The farm market for tractors is supplied by comparatively few manufacturers who have a competitive product market and who buy from suppliers having, in economic theory, the market structure of an oligopolistic environment. In this situation manufacturers, in theory, maximize net revenues by adjusting production to established prices; at least it can be said that production and prices are determined simultaneously. Farmers' purchases of tractors are made on the basis of the established prices. If, in the aggregate, they buy substantially more or less than the anticipated number, a review of production schedules would be in order.

15/ A note on the interpretation of this model with regard to this problem appears in appendix II. This appendix also includes a note on multicollinearity as it may affect the results in this analysis.

16/ This relationship is consistent with adjustment models that consider the stock and flow of investments. A complete discussion of this model appears in Matthews (20).

This equation was specified three different ways to test several hypotheses concerning relationships between $H_{(t)}$ and the independent variables. All these specifications assume that either the natural numbers or the logarithmic values of the variables are additive. First, the dependent variable and all of the independent variables were expressed in logarithms.17/ Second, the dependent variable was expressed in natural numbers with all of the independent variables expressed in logarithms. Third, the dependent variable was again expressed in natural numbers, but with some of the independent variables expressed in natural numbers and the others in logarithms.18/ None of these specifications include all linear relationships because it is not realistic to assume constant quantity changes in the dependent variable associated with changes in all of the independent variables.

In the first formulation of the problem, positive coefficients less than 1 are indicative of decreasing marginal purchases; those equal to 1 show constant marginal purchases, and those greater than 1 show increasing marginal purchases (2).19/ Negative coefficients, regardless of size, show decreasing marginal purchases.

The second and third specifications of the problem only allow for constant and decreasing marginal purchases. They do not allow for increasing marginal purchases that are inconsistent with the principle of diminishing returns.

Consideration of Lagged Values

Inclusion of lagged values in addition to unlagged values for each of the independent variables provides estimates of the delayed effects of each of the independent variables on horsepower purchases.

17/ This kind of specification is for a unique type of joint relationship where changes in the dependent variable associated with changes in any of the independent variables are constant percentages. More generally, joint relationships allow changes in the dependent variable to vary with changes in any or all of the independent variables. Ezekiel and Fox present a more complete discussion of joint relationships (9). Johnston gives a mathematical presentation of the same subject (17).

18/ There was no attempt to include a logarithmic dependent variable with any linear independent variable because this would most certainly be an incorrect specification of the problem. A positive regression coefficient would necessarily have to be interpreted to mean that for an additional unit increase in the independent variable, the dependent variable would increase at increasing rates. For example, with additional increases in crop production, purchases of tractors would have to increase at increasing rates.

19/ Although estimates of tractor purchases must increase for each additional unit increase in the independent variables, each of the successive increases in purchases might either be slightly less, stay the same, or become greater. More specifically, for a unit percentage increase in each of the independent variables, such as crop production, it was assumed that the associated purchases of tractor power changed by a constant percentage which is equal to the regression coefficient.

It shows the adjustment due to the influence of each variable that would be made over a 2- or 3-year period. The short- and long-run elasticities of demand with respect to each of the independent variables are easily calculated from this solution. No assumption is made about the form of the distribution of the lag. For any two independent variables, each having two lagged values, the general regression equation is:

$$(2) \qquad H = a_o + a_1X_{1(t)} + a_2X_{2(t)} + a_3X_{1(t-1)} + a_4X_{2(t-1)} + a_5X_{1(t-2)} + a_6X_{2(t-2)} + u$$

For horsepower purchases as a function of six independent variables, this would be:

$$(3) \qquad H_{(t)}=a_o + \sum_{k=1}^{18}\sum_{j=1}^{6}\sum_{i=o}^{2}a_kX_{j(t-i)} + u$$

where

$H_{(t)}$ = horsepower purchased in period t.

$X_{j(t)}$ = the independent variable associated with t.

$X_{j(t-1)}$ = the independent variable associated with t-1, etc., and

a_k = the regression coefficients.

The regression coefficients for equation 3 in this report are determined in two separate steps.20/ First, results from the statistically fitted functional relationship (equation 1) are used to derive statistically the simultaneous influence of all lagged variables for separate time periods, as shown in the next section (equation 4). Second, results from equations 1 and 4 are used to calculate mathematically the short-run and long-run regression coefficients for all lagged variables as described below (equations 5 and 6).

Combined Influence of All Lagged Variables

The lagged model (equation 4) that shows the influence of all lagged variables combined for separate time periods has independent variables comprising estimates of horsepower purchases derived from the solution of the functional relationships (equation 1) described earlier.

20/ The general procedure for solving this kind of a problem would be to express purchases of tractor power as a function of horsepower on farms last year, crop production, a price index, size of purchases, age of tractors, and number of farms (22). Each variable would be included for a specified number of earlier years. With this procedure, the size of the problem expands rapidly with an increase in the number of factors or lagged time periods. For the six factors considered here in only two lagged time periods, the problem would include 18 variables. The magnitude of this problem prohibits consideration of more than one or two factors in any time series analysis. Use of this general procedure has been confined to estimating demand with lagged values of the economic variables. Even then, the multicollinearity among the lagged values is often so high that the method may only provide erratic regression coefficients for the lagged values and the standard errors may be unacceptable.

In equation 1, actual horsepower purchases were expressed as a function of all the independent variables considered in the present time period. Here, the actual horsepower purchases are expressed as a function of the estimated purchases derived from equation 1 for a specified number of earlier years. For one earlier year this would be:

$$H = b_0 + b_1 \hat{H}_{(t)} + b_2 \hat{H}_{(t-1)} + u \tag{4}$$

where

H = actual horsepower purchased.

$\hat{H}$ = estimated horsepower purchased.

The solution of this equation yields the influence of all variables together in the preceding time periods. In addition, the constant term provides an overall effect associated with the variables in combination with each other.

Influence of Each Lagged Variable

The final distributed lag equation which is mathematically derived shows the influence of each variable for all of the time periods considered.21/ It is the result of substituting a form of equation 1, where only two independent variables are considered, into equation 4, and then simplifying to the form of equation 2. For two variables and one earlier time period, the derivation would be:

$$H = b_0 + b_1\{a_0 + a_1X_{1(t)} + a_2X_{2(t)}\} + b_2\{a_0 + a_1X_{1(t-1)} + a_2X_{2(t-2)}\} + u \tag{5}$$

Expanding the terms in parentheses and combining the constant terms and the regression coefficients, we have: 22/

21/ The association is often statistically significant with this formulation of the problem when indeed it would be difficult to measure this association for several, much less for all, of the independent variables in a conventional distributed lag problem with the seven variables considered here.

22/ Both procedures give the same estimates for purchases of tractor power, but the description of each problem provides different information. The general procedure which has been used gives regression coefficients for each of the independent variables for all of the time periods considered, along with their associated standard errors and the multiple correlation coefficient. The procedure used here gives one regression coefficient for each of the independent variables and one for each of the time periods considered, along with the associated standard errors and multiple correlation coefficients. By multiplying the coefficients of each independent variable by the coefficient related to that time period, we get the regression coefficients generally derived for each independent variable for one or more earlier time periods.

(6) $$H_{(t)} = d_0 + d_1X_{1(t)} + d_2X_{2(t)} + d_3X_{1(t-1)} + d_4X_{2(t-1)} + u$$

which is the same type of formulation as equation 2.

This equation provides for both short-run and long-run elasticities of demand with respect to each of the independent variables. The short-run elasticities are those related to each of the variables in time period (t). The long-run elasticity for any independent variable is the sum of the short-run elasticities for the same variable over all the time periods considered.

RESULTS

The results of fitting selected demand equations to the data are shown in the next three sections. The first section includes the regression coefficients for variables considered in only one time period. No lagged variables are included. The second includes the regression coefficients showing the influence of time on all variables combined. The third combines the results of the first two to show the influence of each of the variables over time. These results were not influenced unduly by using the size of tractors purchased for calculating the dependent variable (horsepower purchases) and then including the size of tractors purchased as an independent variable. The coefficient of determination (r^2) between horsepower purchases and size of tractors purchased was only 0.42, whereas all of the fitted equations had coefficients of multiple determinations (R^2) of 0.90 or more.

Demand for Tractor Horsepower Purchases

The results of fitting three different equations that did not include lagged values are shown in table 1. These equations assume an association between horsepower purchases and each of the independent variables for each year. They do not assume any association between horsepower purchases this year and each or any of the independent variables in earlier years. The association of any of the independent variables with horsepower purchases assumes that the other independent variables remain constant.

In general, all the independent variables were related to horsepower purchases in the manner anticipated. An increase in the price ratio (ratio of tractor prices to prices received for farm products sold) which reflects changes in income and changes in the real price of tractors was associated with fewer tractor purchases. Changes in the variables related to depreciation, personal preferences, and technology had an overall positive association with more tractor purchases. Additional tractor purchases associated with increases in horsepower on farms, crop production, and average size of tractors purchased were partly offset by the negative influence of increases in the average age of tractors and the lower positive influence of the reduction in the number of farms.

More tractor purchases are normally expected as the stock of tractors on farms or crop production increases. It is not as obvious that tractor purchases are associated positively with the size of tractors or the number of farms. It is logical that as the size of purchased tractors increases, the horsepower purchased will also tend to increase. With an increase in size of tractors, farmers will tend to buy slightly fewer tractors that are considerably larger and will be purchasing more total horsepower.

Table 1.--Regression coefficients and related data associated with tractor horsepower purchases, United States, 1921-41 and 1947-62 1/

Problem number	Horsepower on farms, January 1	Crop production	Ratio of tractor price to prices received	Average size of tractors purchased	Average age of tractors on farms	Number of farms, January 1	R^2	Constant	Standard error of estimate (million)
1 2/ ------------	3/0.828 (.219) **	3/0.685 (.765)	3/-2.642 (.379) **	3/7.317 (1.890) **	3/-1.783 (.560) **	3/9.560 (2.454) **	90	-13.312	4/1.95
2-------------	3/12.877 (1.557) **	3/4.749 (5.428)	3/-12.865 (2.867) **	3/18.066 (13.415) *	3/-14.985 (3.977) **	3/37.023 (17.419) **	95	-40.921	.91
3-------------	.150 (.017) **	.083 (.030) **	3/-13.934 (2.581) **	3/23.504 (13.041) **	3/-7.717 (3.397) **	3/98.308 (18.696) **	95	-85.823	.88

* Coefficients significant at the 90-percent level.
** Coefficients significant at the 95-percent level.
1/ Numbers in parentheses are standard errors of the regression coefficients.
2/ Dependent variable expressed in logs.
3/ Independent variable expressed in logs.
4/ Standard error of estimate calculated for natural numbers of dependent variable.

Likewise, horsepower purchases decline with a reduction in the number of farms. Fewer farms are associated with larger farms, and the tractors on farms going out of business move to the farms that remain. Thus, there is the same quantity of horsepower available for use by fewer farmers. However, it is generally recognized that the fewer and larger farms that remain will require less horsepower per acre than the smaller farms that went out of business.

Equation 1

In the first equation, where all of the variables are in logarithms, 90 percent of the variation in tractor horsepower purchases is explained by the variation in horsepower on farms, crop production, the ratio of the price of tractors to prices received for farm products sold, size of tractors purchased, age of tractors on farms, and number of farms. Except for crop production, all of the coefficients are statistically significant at the 95-percent level.

This type of problem formulation assumes that each 1-percent change in any independent variable is associated with constant percentage changes in horsepower purchases that are equal to the regression coefficient. That is, every 1-percent increase of horsepower on farms from the preceding year was, on the average, associated with an 0.8-percent increase in horsepower purchases, provided the following remained constant: (1) Crop production, (2) ratio of tractor prices to prices received, (3) size of tractors purchased, (4) age of tractors on farms, and (5) number of farms. Also, 1-percent changes in the ratio of tractor prices to prices received and in the average age of tractors on farms were associated with 2.6-percent and 1.8-percent changes, respectively, in horsepower purchases in the opposite direction. Similarly, 1-percent changes in the size of tractors purchased and in the number of farms were associated with 7-percent and 10-percent changes in purchases of tractor horsepower in the same direction. The effect of crop production on horsepower purchases is not certain.

This formulation of the problem indicates decreasing marginal purchases for larger quantities of horsepower on farms, and for higher levels of crop production.23/ This is consistent with economic theory. The problem also shows marginal purchases increasing for increases in average size of tractors purchased and in the number of farms. That is, during the period of analysis with this problem formulation, it is suggested that purchases increased at increasing rates with the purchase of larger sizes of tractors or with increases in the number of farms. Lower farm numbers would contribute to decreasing purchases at a decreasing rate. Also, for increases in the price variable and in the age of tractors on farms, purchases were reduced at decreasing rates.

Equation 2

In the second equation, where the dependent variable is in natural numbers and the independent variables are all in logarithms, 95 percent of the variation in tractor horsepower is explained by the variation of the independent variables. With this formulation of the problem, the average size of tractors and crop production are not significant at the 95-percent level.

23/ For this equation, positive coefficients less than 1 indicate decreasing marginal purchases; those equal to 1 show constant marginal purchases; and those greater than 1 show increasing marginal purchases. Negative coefficients, regardless of size, show decreasing marginal purchases.

In this equation, each 1-percent change in any of the independent variables was associated with a constant quantity change in horsepower purchases approximately equal to 0.00434 times the regression coefficient.24/ These independent variables are assumed to be nonlinearly related to the dependent variable; purchases will increase at decreasing rates for unit increases in the independent variables. This specification allows for decreasing purchases for additional units of all the independent variables.

This equation indicates that during the period of study each 1-percent increase in tractor horsepower on farms was associated with an increase in purchases of about 56,000 (0.00434 times 12.877 million) tractor horsepower. Likewise, a 1-percent decrease in the number of farms was associated with 160,000 (0.00434 times 37.023 million) fewer horsepower purchases. Also, each 1-percent increase in the price ratio and the average age of tractors was associated with 56,000 and 65,000 fewer horsepower purchases, respectively. Conversely, a 1-percent decrease in the price ratio and the average age of tractors was associated with 56,000 and 65,000 more horsepower purchases, respectively. The effect of crop production and the average size of tractors is not certain with this formulation of the problem.

Equation 3

In the third equation, the dependent variable and two of the independent variables (horsepower on farms and crop production) are expressed in natural numbers. The other independent variables are expressed in logarithms. With this formulation, 95 percent of the variation of the dependent variable is explained by variations of the independent variables and all of the regression coefficients are significant at the 95-percent level.25/

24/ A 1-percent increase in any number is equal to a 0.00432-increase in the common logarithm of that number. Also, a 1-percent decrease in any number is equal to a 0.00436-decrease in the common logarithm of that number. On the average, a 1-percent change in any number is equal to a 0.00434-change in the common logarithm of that number.

25/ A still higher percentage of the variation in tractor horsepower purchases is explained by variations in the independent variables when time periods are multiples of years rather than single years. In this formulation of the problem, horsepower purchases and crop production are summations of annual estimates. Values of the other variables are averaged for the period. By using 2-year estimates rather than annual estimates, the explained variation increased from 95 to 98 percent. The explanation of more variation was accompanied by a reduction from 15 percent to 11 percent in the ratio of the standard error of estimate to the mean values of the dependent variable. As might be expected where purchases are 2-year estimates, the regression coefficients almost doubled in size because of the continuing trends in the independent variables. The regression coefficients retained almost the same degree of accuracy as for the annual estimates except for the horsepower on farms, but the coefficient was still highly significant. The generally greater positive influence of horsepower on farms, crop production, size of tractors purchased, and number of farms were offset by the higher negative influence of the ratio of tractor prices to prices received and the average age of tractors.

In this equation each unit increase in horsepower on farms and in crop production was associated with a constant quantity change in horsepower purchases equal to the regression coefficient. That is, these variables were assumed to be linearly related to the dependent variable. On the other hand, each 1-percent change in the price variable, size of tractors purchased, age of tractors on farms, and number of farms was associated with a constant quantity change in horsepower purchases equal to approximately 0.00434 times the regression coefficient. Here these four independent variables are assumed to be nonlinearly related to the dependent variable; purchases increase at decreasing rates for unit increases in the independent variables.

The third equation shows that each 10 additional horsepower on farms and each 10-point rise in the index of crop production was associated with about 1.5 and 0.8 more horsepower purchased, respectively. In addition, each 1-percent increase in the average size of tractors purchased was associated with about 102,000 more horsepower purchases. Also, 1-percent increases in the real price of tractors and in the average age of tractors were associated with about 67,000 and 37,000 fewer horsepower purchases, respectively. And each 1-percent decrease in farm numbers would be associated with 427,000 fewer horsepower purchases.

This equation provides reasonable estimates of the relationships between the dependent and independent variables during the study period. It is the equation used for estimating purchases as a function of estimated values for this and earlier years. It is used here for making a projection to 1970. The residuals of this equation appear to have more uniform variance throughout the period studied than the first two equations.26/ Serial or autocorrelation is not indicated by the Durbin-Watson d statistic in any of these equations.27/

Combined Influence of All Lagged Variables

With these data, any consideration of the combined effect of all the independent variables in earlier time periods provides improved estimates of tractor horsepower purchases. Generally, estimates for individual years are improved if the standard error of all the estimated values is lowered. The combined influence of earlier time periods on horsepower purchases is shown in table 2. These equations are derived from their counterparts in table 1.

The influence of earlier time periods reduced the standard error up to a third, but explained about the same variation of the dependent variable associated with changes in the independent variable (tables 1 and 2). Generally, the errors were reduced throughout the whole period; however, they were reduced more for the larger estimated values in recent years than for the smaller values in earlier years. That is, the original

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26/ Such residuals are said to be homoscedastic. A more elaborate discussion appears in Mood(21).

27/ In the first two equations, negative serial correlation is insignificant at the 5-percent level, but the results are inconclusive for positive serial correlation. In the third equation, positive serial correlation is insignificant at the 5-percent level, but the results for negative serial correlation are inconclusive. Lack of evidence of autocorrelation does not specify its absence (17). Some of the independent variables may still be related to their lagged values.

Table 2.--Regression coefficients with their standard errors, multiple correlation, and the standard error of estimate associated with the influence of earlier time periods on tractor purchases, 1921-41 and 1947-62 1/

Equation form 2/	Regression coefficients of horsepower purchases			Multiple determination coefficient (R^2)	Standard error of estimate (million)	Constant
	This year	Last year	Year before			
1	0.906 (.101)	0.123 (.101)	---	90	1.32	-0.017
1	.963 (.114)	.156 (.106)	-0.101 (.095)	90	1.32	-.012
2	1.138 (.209)	-.125 (.201)	---	95	.86	-.122
2	1.108 (.210)	.137 (.316)	-.230 (.214)	95	.86	-.155
3	.933 (.153)	.070 (.149)	---	95	.85	-.025
3	1.008 (.157)	.246 (.183)	-.246 (.156)	96	.83	-.074

1/ Numbers in parentheses are the standard errors of the regression coefficients.

2/ Equation forms refer to the corresponding problem numbers in table 1. Here, as in table 1, all variables in equation form 1 are expressed in logarithms. However, in contrast, here all variables in equation forms 2 and 3 are linear. This is necessary to give distributed lag values for the independent variables consistent with the proposed distributed lag model.

regression equations in table 1 had larger variances in the later years, while the equations considering the influence of earlier time periods in table 2 had a more constant variance throughout.

The influence of earlier time periods also reduced the serial or autocorrelation of the residuals. Results from using the Durbin-Watson d statistic to test for serial correlation showed both insignificant negative and positive serial correlation.

Certainly, estimates of tractor horsepower purchases could be improved by considering the influence of earlier time periods. An apparent effect would be to lessen the changes between years. In the regression model described above, changes between years, when the influence of earlier time periods was considered, were only about half as large as those when they were not considered. 28/ The new estimates are actually fluctuating less around longer time trends.

In general, when one earlier time period was considered, about 10 percent of the influence of each variable was related to the earlier time period. When two earlier time periods were considered, the sign of the earliest time period was no longer consistent with the expected values in the formulation of this problem. The immediate thought is to discard these values in this problem. 29/ Changes in the sign may well mean that the influence of the earlier time period is not measurable since the change in sign suggests an erratic behavior. However, this may also suggest a cyclical nature in tractor purchases. A 4- or 5-year cycle in purchases is consistent with a negative regression coefficient for variables lagged 2 years.

Influence of Each of the Lagged Variables

The distributed lag equations showing the influence of each of the independent variables for the preceding year are shown in table 3. The equations state the relation of each of the independent variables for both this year and last year with tractor purchases. They are interpreted the same as any other regression coefficient. For example, the first

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28/ With the lesser fluctuation associated with influences from earlier time periods, one might expect the direction of estimated year-to-year changes to be different from those in which the influence of earlier time periods is not considered. Except for 3 years, the directional changes between years were the same whether or not the influence of earlier time periods were considered. In the 3 years where directional changes differed when earlier time periods were considered, changes between years were small in either direction. However, in each of these 3 years, the directional changes of the regression model that did not consider the influence of earlier time periods agreed with actual changes between years.

29/ Tinbergen suggests that only lagged values be added until the signs become erratic and no longer make sense (26). Both Nerlove and Tinbergen suggest that these erratic signs are usually a result of the intercorrelation of lagged explanatory variables. Obviously, a high standard error of the coefficients may necessitate the exclusion of more time periods, even when the signs are still consistent with logic.

Table 3.--Calculated regression coefficients for present and lagged values of the variables associated with tractor purchases for equations 1 and 3 in table 1

Variable	Equation 1 1/	Equation 3 2/
Horsepower on farms on Jan. 1--		
This year	0.750	0.140
Last year	.102	.011
Crop production--		
This year	.621	.077
Last year	.084	.006
Ratio of tractor price to prices received--		
This year	-2.394	-13.000
Last year	-.325	-.975
Average size of tractors purchased--		
This year	6.629	21.929
Last year	.900	1.645
Average age of tractors on farms--		
This year	-1.615	-7.200
Last year	-.219	-.540
Number of farms--		
This year	8.661	91.721
Last year	1.176	6.882
Constant term	-13.715	-86.106

1/ Calculated by multiplying the coefficients of the first equation form 1 in table 2 times the regression coefficients of equation 1 in table 1.
2/ Calculated by multiplying the coefficients of the first equation form 3 in table 2 times the regression coefficients of equation 3 in table 1.

two coefficients in equation 3 of table 3 are interpreted as follows: Each unit increase in horsepower on farms this year and last year was associated with 0.140- and 0.011-unit increases in horsepower purchases, respectively. This regression equation has 12 independent variables instead of 6. These equations estimate the same purchases as the lagged equations in the preceding section that combine the influence of all lagged variables (table 2). In addition, these show the greater importance of the present values for each independent variable and the much lesser importance of the lagged values. The equations were derived by calculating the constant term and by multiplying the regression coefficients in table 1 times their counterparts in table 2, as previously described.

Elasticities of Demand

The elasticities of demand with respect to each of the independent variables vary with the type of problem formulation (table 4). The elasticities of equation 1 are averages for the period studied and are the same for all values of each of the independent variables.

The elasticities of equation 3 vary with different values of the independent variables. 30/ The elasticities at both mean values and recent values for horsepower on farms, crop production, and the number of farms are higher than the constant elasticities in equation 1. Also, elasticities of the price variable, average size of tractors, and the average age of tractors in equation 3, at both mean and recent values, are less than in equation 1.

In general, the elasticities of equation 1 indicate that about 89 percent of the adjustment in tractor purchases related to the independent variables are made in the short run. Equation 3 shows that about 93 percent of the adjustment is made in the short run.

The elasticity of demand with respect to horsepower on farms has increased in recent years as compared with average values for the period. The elasticity with respect to crop production remained about the same. But the elasticity of the real tractor price, average size of tractors purchased, average age of tractors on farms, and number of farms have decreased between 20 and 25 percent in recent years. The elasticity of demand for new tractors, with respect to the real price of tractors (ratio of tractor prices to prices received for farm products sold), is negative and is greater than unity. In recent years, it was estimated to be about -1.7 in the short run and -1.8 in the long run. The long-run estimates are only slightly higher than those of Griliches (13). However,

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30/ Elasticities of horsepower on farms and crop production are found by multiplying a specified value of the independent variable times the regression coefficient of that variable, and then dividing by the value of horsepower purchases estimated for corresponding values of all of the independent variables. Elasticities of the ratio of tractor prices to prices received, average size of tractors purchased, average age of tractors on farms, and number of farms are found by dividing the regression coefficient of the variable under consideration by the value of horsepower purchases estimated for specified values of all independent variables.

Table 4.--Short- and long-run elasticities of demand with respect to all the independent variables in equations 1 and 3 in table 3

Variable	Equation 1[1/]		Equation 3[2/]			
			Mean values		Recent values	
	Short-run	Long-run	Short-run	Long-run	Short-run	Long-run
Horsepower on farms, Jan. 1	0.8	0.9	1.6	1.7	3.0	3.2
Crop production	.6	.7	1.1	1.2	1.1	1.2
Ratio of tractor prices to prices received	-2.4	-2.7	-2.2	-2.4	-1.7	-1.8
Average size of tractors purchased	6.6	7.5	3.7	4.0	2.9	3.1
Average age of tractors on farms	-1.6	-1.8	-1.2	-1.3	-.9	-1.0
Number of farms	8.7	9.8	15.6	16.8	11.9	12.8

1/ Both the dependent and independent variables are logarithmic.
2/ The dependent variable and the first two independent variables are in natural numbers. The last four independent variables are logarithmic.

short-run price elasticities differ widely from those found by Griliches because of differences in estimated time required to make adjustments. While it was found here that between 80 and 90 percent of the adjustment was made in the short run, Griliches found that only about 20 percent of the adjustment was made in this period. These differences are primarily due to the different procedures used for estimating long-run and short-run coefficients. The long-run coefficients described here are sums of the coefficients considering each of the independent variables in earlier time periods as shown previously. Griliches' long-run coefficients are calculated from an adjustment coefficient which describes the difference between the desired and the actual stock of tractors.

PROJECTED PURCHASES FOR 1970

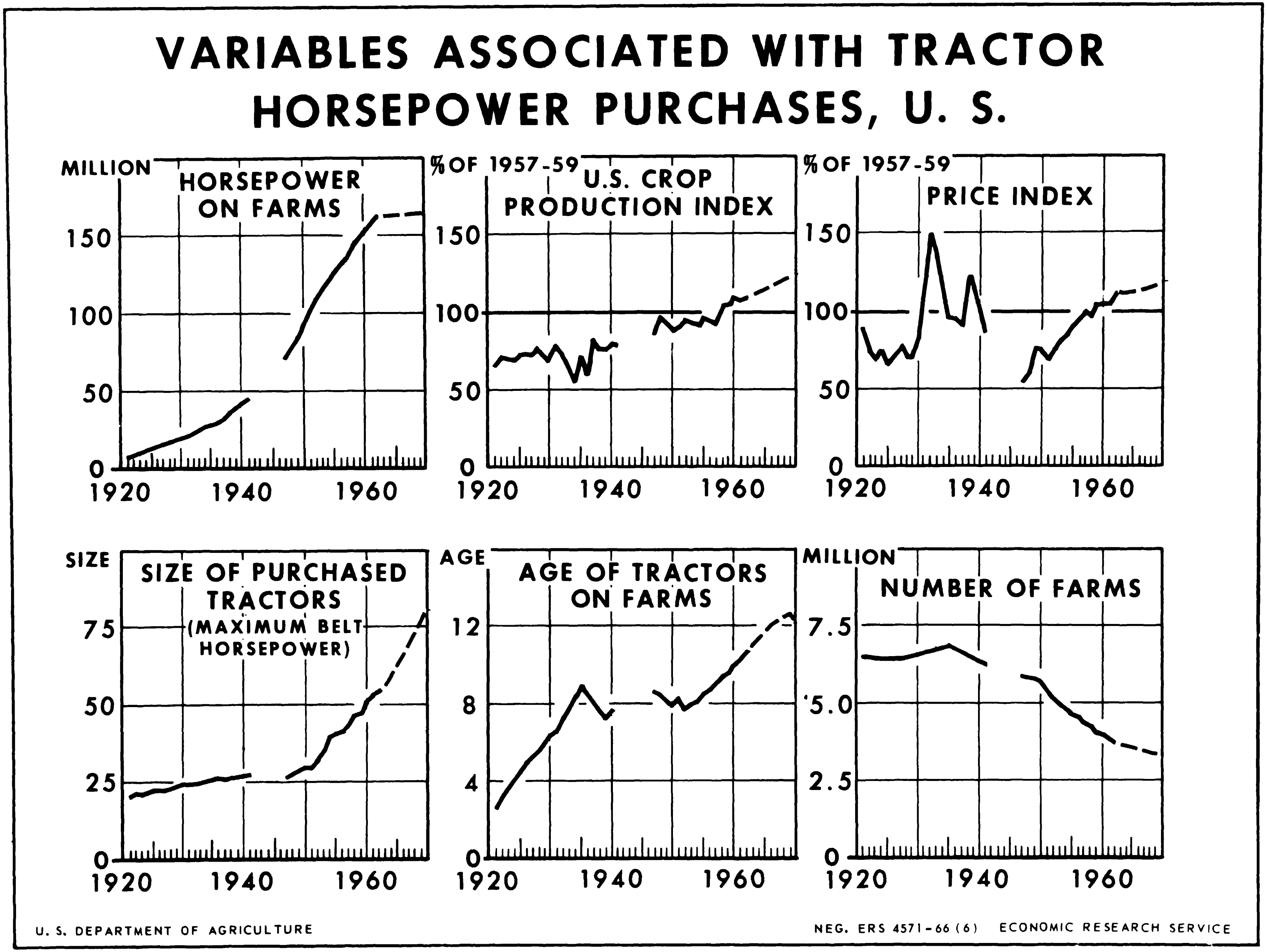
Equation 3 in table 1 is used here to project average purchases for several years in some future time period. 31/ This equation has statistically significant regression coefficients and will more readily allow for diverse movements of the independent variables. 32/

Purchases for 1970 are projected at about 8 million horsepower--up from about 7½ million in 1962--if the following conditions prevail: (1) Crop production increases about 15 percent (index of crop production, 1957-59=100, up from 108 to about 124); (2) ratio of tractor prices relative to prices received for farm products sold increases about 7 percent (index of tractor prices divided by the index of prices received, 1957-59=100, up from 110 to 118); (3) size of tractors purchased increases from 54.5 to 80.0 horsepower; and (4) number of farms decreases from 3.7 million to 3.3 million (figs. 6 and 7). A wide variation in projected purchases results from considering relatively small differences in the independent variables. However, changes in these variables would tend to offset each other. The number of tractors sold in 1970 will be about 100,000 units, if the average size of tractors purchased increases to about 80 horsepower.

Projections for crop production, price ratios, size of purchases, and number of farms were made on a year-to-year basis so that values for average age of tractors and horsepower on farms could be calculated each year. The calculated values for the latter two variables were used in the regression equation to estimate purchases. This procedure was necessary for keeping relationships consistent when moving through time. If either the horsepower on farms or the average age of tractors were projected rather than calculated, it would be unlikely that purchases would have been such that the calculated values for these two variables agreed with projected values.

31/ Equation 3 in table 3 might better be used for projecting year-to-year purchases of tractor horsepower. This equation assigns weights to estimated purchases for the last 2 and/or 3 years.

32/ The effect of individual variables takes on additional importance for making projections. Here the combined results of the standard error of estimate and the standard errors of the regression coefficients are considered to be more important than the multiple correlation coefficient.


VARIABLES ASSOCIATED WITH TRACTOR HORSEPOWER PURCHASES, U. S.
MILLION
HORSEPOWER ON FARMS
% OF 1957-59
U.S. CROP PRODUCTION INDEX
% OF 1957-59
PRICE INDEX
SIZE
SIZE OF PURCHASED TRACTORS (MAXIMUM BELT HORSEPOWER)
AGE
AGE OF TRACTORS ON FARMS
MILLION
NUMBER OF FARMS
0
50
100
150
0
25
50
75
0
4
8
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1920
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1960
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ECONOMIC RESEARCH SERVICE

Figure 6

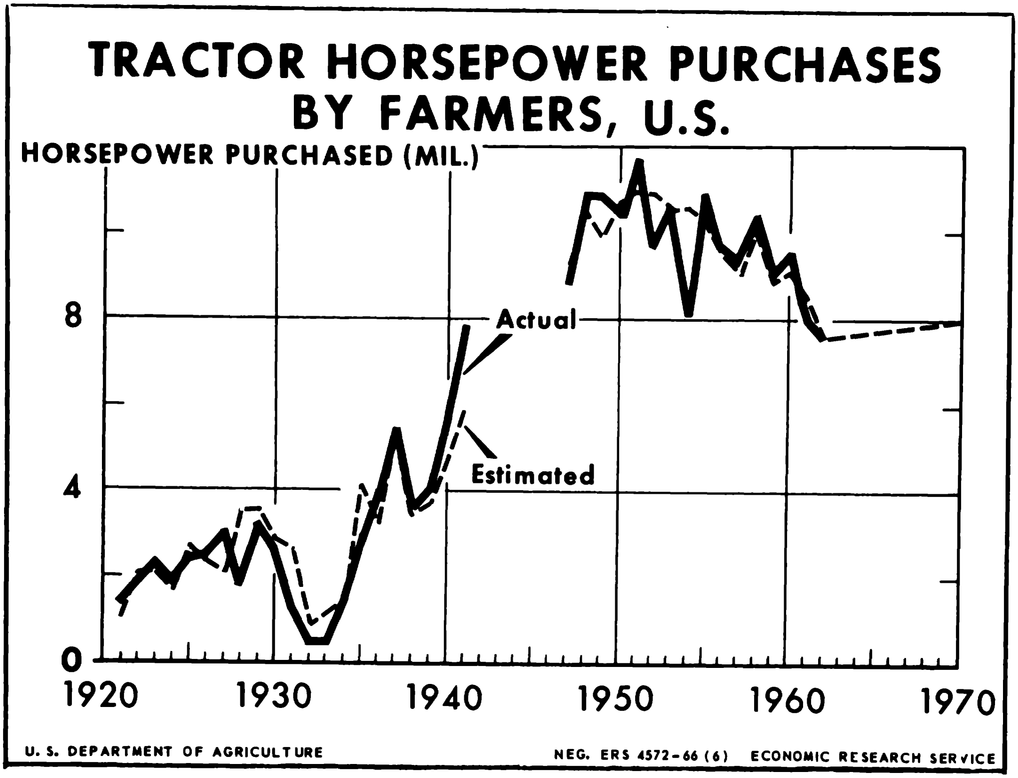

Figure 7

The number and size of tractors discarded were estimated for 1962-70 so that the horsepower on farms in the beginning of the year and the average age of tractor could be calculated. The number of tractors discarded parallels recent estimates used in calculating changes in tractor numbers on farms. This schedule is based largely on the results of earlier studies and a national farm machinery survey in 1956 (4, 23). No tractors were discarded less than 3 years after purchase(fig.8). They were discarded at an increasing rate between the 3rd and the 19th year. The percentage of tractors discarded continued to increase, but at a decreasing rate, until all of them were discarded at 32 years of age.

If applied to an equal number of purchases over time, this discard schedule would result in an average age for farm tractors of 11 years. Also, the average age of tractors discarded, sometimes called the average service life, would be 24 years. In 1962, the average age of tractors was about 11 years and the average age of tractors discarded was 15 years. By 1970 the average age of tractors would increase to almost 13 years and the average age of discards decrease to less than 14 years. The average age of tractors on farms would increase between 1962 and 1970 because of the unusually large purchases made between 1947 and 1953. The greater number of discards now coming from these earlier purchases also tends to reduce the average age of the tractors being discarded. Since the age of tractors is affected by the larger purchases, in the earlier years the age was somewhat less than what would be normally expected, and now in later years the age of tractors is more than what would be expected.

The discard schedule applied to an increasing size of tractors over time would result in discard of a higher average horsepower for a given age of tractor. Also, the average service life in terms of horsepower would be lowered.

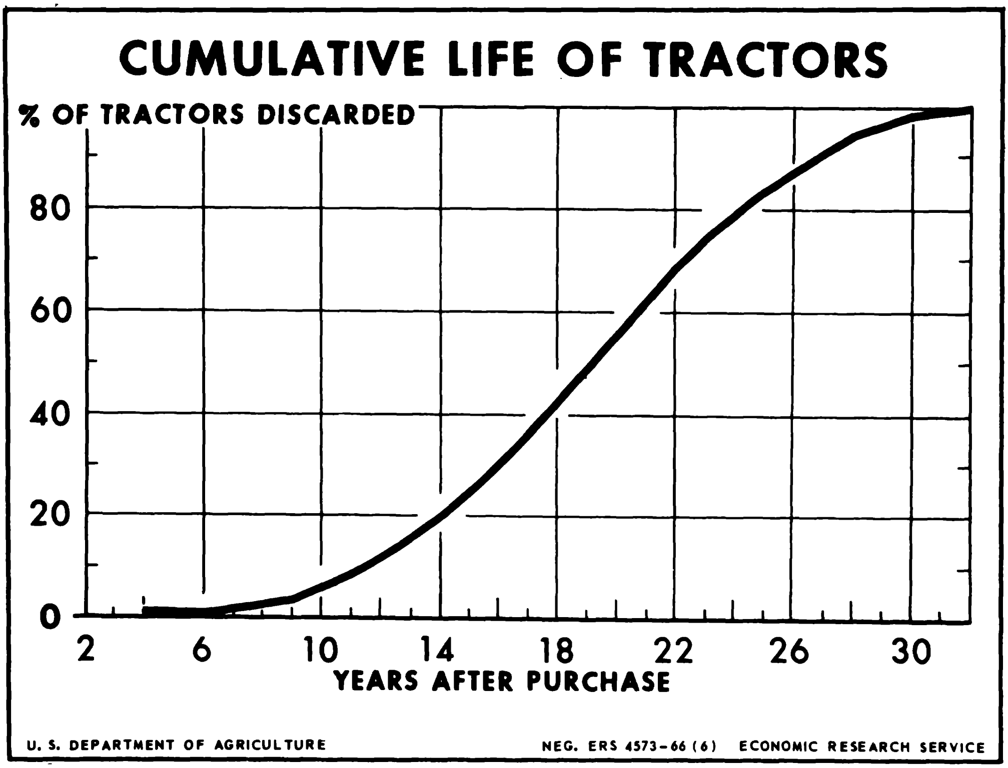

Figure 8

The projections in tractor horsepower would mean a reduction in the number of tractors on farms. Tractor numbers on farms would decrease from 4.7 million tractors on farms in 1962 to 3.65 million tractors in 1970.

Among other things and in addition to the purchases described above, this reduction assumes that discards will be made as they were in the middle 1950's when the average life of tractors would have been about 11 years if the same number of tractors were bought each year.

The projections of this regression equation should be considered as an additional source of information. These results might be weighted against other independent estimates. From these separate sources a final estimate is made, largely a judgment estimate, based primarily on the relative merits of the separate estimates.

Farm power will continue to be an important factor in agricultural production. It is indeed doubtful that changes in the next 40 years will be of the same magnitude as those in the preceding 40 years. In future years it is likely that total farm power available for use will increase at a much lower rate. Although tractor power has increased from about an eighth of the total power in 1920 to 40 percent in recent years, it is expected to level off while other sources of power will continue to increase. Thus, tractor power will be a smaller proportion of total farm power in the future than it has been in the recent past.

CONSIDERATIONS IN USING REGRESSION MODELS

Despite the care used in specifying the model and selecting the variables to be included, there are always possibilities of changes in demand over time for which no adjustments were made. Use of data not properly adjusted for changes in demand will result in fitting equations to points on separate demand schedules rather than points on a single demand schedule.

Also, an aggregate analysis such as this necessarily ignores the individual actions described earlier for many homogeneous groups of farms. A change in the structure, that is, a change in the relative importance of these specified groups, throughout the period of study necessarily complicates the interpretation of results. However, the success or lack of success in explaining aggregate changes in the past may not be associated with a real understanding of internal forces bringing about these changes. The analyst can only hope that the internal structure remains stable or continues to change in a predictable pattern.

Although improperly fitted curves cannot be used to give the exact elasticities, they are useful in describing what has happened in some earlier period and they provide a means of projecting to some later period. Regression models per se were designed to estimate and attempt to explain historical relationships. They were also considered to serve as a guide in explaining some forthcoming events if, and only if, none of the variables extended beyond the range of the data in the base period. This means not only that values of individual variables cannot exceed their range, but also that any combination of the variables must stay within past relationships.

In considering the relative merits of any regression equation used to make projections, the researcher must always remember that he is almost certain to extend the model beyond the range of the original data. In doing this, he assumes that basic relationships in the future will continue as they were when they were included in the model during some base period. In effect, if the curves have been fitted to points on one demand schedule, use of this model for making projections assumes that the demand will remain the same over time. That is, the demand with respect to each independent variable will stay the same or will change over time as it did during the base period. However, if the curves have been fitted to points on separate demand schedules, the implication should be that demand will continue to change as it had in the past and that farmers will purchase more or fewer tractors in the future much the same as in the past.

In terms of the model used in this study, the projections assume that purchases of tractor horsepower in future years will, on the average, be influenced proportionately as they were during 1921-41 and 1947-62 for any future changes in the following: (1) Horsepower on farms, (2) crop production, (3) ratio of tractor prices to prices received for agricultural products sold, (4) size of tractors purchased, (5) age of tractors on farms, and (6) number of farms. In essence, the structural relations are held constant while alternative values of the independent variables may be considered. The influence of some of the independent variables in real terms will actually become less as the variables become larger in size. But we are still assuming that changes of the independent variables affect the dependent variable in the future as they did in the base period of study, even though the variables themselves may change.

The large weight assigned to changes in farm numbers weakens this model for making projections into the future. The changes in farm numbers also affect changes in farm structure. Thus, the influence of changes in technology is included in changes in the number of farms. Since the number of farms has been reduced sharply during the period of study and since it will most likely continue to decline, the influence of farm numbers becomes exaggerated. It is unlikely that changes in technology associated with changes in farm numbers will be the same in 20 or even 5 years from now. Present trends suggest that these changes in technology associated with the number of farms will become less important. Therefore, effects of fewer farms in the coming years will be to understate horsepower purchases.

Furthermore, the recent expansion of U. S. food aid to needy countries, if continued over a period of years, would affect the variables related to changes in technology. Thus, the newly projected values for the independent variable and the calculated projections of tractor purchases in this report would be slightly higher.

APPENDIX I: THE VARIABLES

Values for tractor horsepower purchases and variables assumed to be related to purchases are shown in table 5 for the study period. This section elaborates on the source and development of the variables.

Tractor Horsepower Purchases

Tractor horsepower purchased for farm use is the product of the number of wheel and crawler tractors purchased and the average size of purchases, but does not include garden tractors. The number of tractors purchased by farmers was estimated by Paul Strickler of the Economic Research Service on the basis of tractor shipments published in the Current Industrial Reports (31).

Size of Tractor Purchases

The average size of tractors purchased in each year is the average maximum belt horsepower. It is estimated from the manufacturers' shipments to dealers by weighting the midpoint of each size group by the number of tractors. It is only in the later years of this study that tractor shipments have been stated as maximum belt horsepower. Adjustments made for the earlier shipments reported as average belt horsepower were generally about 75 percent of maximum belt horsepower. This can be written as:

$$S_{p(t)} = \frac{\sum_{i=1}^{n} N_i \frac{L_i + H_i}{2}}{\sum_{i=1}^{n} N_i}$$

where

$S_{p(t)}$ = average size of purchases in year t.

N_i = number of tractors purchased in size group i.

Table 5.--Variables assumed to be associated with tractor horsepower purchases, United States, 1921-41 and 1947-62

Year	Horsepower on farms, January 1 (million)	Crop production	Ratio of tractor prices to prices received	Average size of tractors purchased	Number of farms, January 1 (million)	Average age of tractors on farms	Horsepower purchased (million)
1921	6.997	65	88	20.6	6.511	2.70	1.339
1922	7.738	70	74	21.0	6.500	3.33	1.995
1923	9.031	70	71	21.3	6.492	3.78	2.300
1924	10.614	69	75	21.6	6.480	4.16	1.966
1925	11.968	72	66	22.0	6.471	4.62	2.486
1926	13.662	73	72	22.3	6.462	4.99	2.520
1927	15.454	72	76	22.6	6.458	5.34	3.006
1928	17.751	75	70	23.0	6.470	5.61	1.886
1929	19.269	73	70	23.6	6.512	6.11	3.233
1930	21.804	69	83	24.0	6.546	6.34	2.640
1931	23.928	77	117	24.3	6.681	6.68	1.336
1932	24.835	73	148	24.6	6.687	7.23	.541
1933	25.067	65	134	24.9	6.748	7.86	.548
1934	25.298	54	111	25.2	6.776	8.41	1.562
1935	26.410	70	95	25.5	6.814	8.66	2.932
1936	28.688	59	95	25.8	6.739	8.51	3.999
1937	31.734	81	91	26.1	6.636	8.15	5.481
1938	35.620	76	121	26.3	6.527	7.57	3.682
1939	38.003	75	114	26.6	6.441	7.34	4.070
1940	41.682	78	101	26.9	6.350	7.64	5.784
1941	44.788	79	87	27.2	6.293	7.59	7.834
1947	70.786	85	55	26.3	5.871	8.69	8.810
1948	76.167	97	61	27.6	5.803	8.49	10.930
1949	84.571	92	77	28.8	5.722	8.18	10.915
1950	92.588	89	75	29.4	5.648	7.98	10.408
1951	101.071	91	69	29.4	5.428	8.11	11.760
1952	108.263	95	75	31.7	5.197	7.78	9.732
1953	114.923	94	81	35.1	4.984	7.96	10.530
1954	121.180	93	84	39.0	4.798	8.10	8.151
1955	126.439	96	90	40.9	4.654	8.48	10.879
1956	133.773	95	95	41.0	4.514	8.76	9.758
1957	139.476	93	99	43.5	4.372	9.07	9.396
1958	144.329	104	96	46.1	4.233	9.38	10.419
1959	149.957	103	105	46.5	4.097	9.66	9.160
1960	153.963	108	105	50.8	3.949	9.97	9.500
1961	158.813	107	105	53.2	3.811	10.35	8.033
1962 1/	162.321	108	110	54.5	3.688	10.72	7.630

1/ Preliminary.

L_i = lowest horsepower in size group i.

H_i = highest horsepower in size group i.

Tractor Horsepower on Farms

Tractor horsepower available for farm use is the product of the number of tractors and the average size of tractors on farms. The number of wheel and crawler tractors on farms is published by USDA (29) and does not include garden tractors. These numbers are closely tied to the agricultural census data. Annual sales of tractors to farmers are added to numbers on hand and estimates of numbers scrapped are subtracted. Rates of scrapping are initially estimated from the length of life as derived from periodic surveys of the number, size, and age of tractors on farms. Adjustments are made in the scrapping rates so that the number of tractors on hand agrees with the number of tractors on farms in census years. Recent estimates of tractors on farms are based on census numbers in 1959. Scrapping rates are adjustments of rates obtained in a nationwide survey by USDA of farm machinery in 1956.

Size of Tractors on Farms

The average size of tractors on farms for each year was derived by multiplying the number of tractors purchased in any given year that are still on farms by the average size of purchases for the given year, and getting a total for similar purchases in the earlier years; this total is then divided by the total number of tractors still on farms for the year in question and for earlier years.

In equation form, this would be:

$$S_{(t)} = \frac{\sum_{i=a}^{t} N_i H_i}{\sum_{i=a}^{t} N_i}$$

where

$S_{(t)}$ = average size in year t.

N_i = number of tractors purchased in ith year that are still on farms in year t.

H_i = horsepower per tractor purchased in the ith year.

a = the oldest year for which tractors still on farms were purchased.

Crop Production

The variable used to measure crop production was the index of all crops in the United States (1957-59 = 100) as published by USDA (29). It is a composite of all agricultural crops and is a weighted aggregate of the constant dollar values for each crop. The weights assigned to each crop varied throughout the period. Constant dollar values with 1935-39 weights were used in the early years, 1920-40; 1947-49 dollars were used in the middle years, 1941-59; and 1957-59 dollars in the latest years, 1960-62. The indexes were spliced between weighting periods by estimating both indexes for several years and then expressing all the earlier years as an index in terms of the latest weighting period.

Price Variables

The price variable is a ratio of tractor prices relative to prices received for farm products sold (constructed index of tractor prices divided by the USDA index of prices received for agricultural products sold, 1957-58=100). It reflects the real change in purchasing power for farm machinery.

The price paid for tractors is the single average of prices paid for tractors in four size groups over time. This price change is nearly the same as that for all farm machinery. The price of the tractors in each size group for individual years reflects changes in quality and quantity, and a shift to larger sizes within each group. Quality changes reflected in price include factors such as better materials that cost somewhat more than the older materials, like nylon as contrasted with cotton in tires. Quantity changes include the many extras that are integral parts of the new tractors, such as hydraulic controls, power steering, and automatic transmissions.

Apparent price increases in this analysis that are actually changes in quality and quantity are generally offset by changes in quality and quantity in the index of prices received for agricultural products sold. Both quality and quantity changes in agricultural products reflect the higher grade of food products being sold.

Indexes of both prices paid for farm tractors and prices received for agricultural products sold are subject to error because of weighting systems of the quantities purchased and sold.

In considering the demand for commodities--whether they be consumption goods or durable goods--prices of competitive inputs should be considered. This would have to include some assumption about new technological innovations, along with associated prices for labor and horses. In terms of this study, demand for tractor horsepower might well be influenced by prices of horses and farm labor. They were not included in this study because it is difficult to assess the importance of changes in the prices of horses and farm labor as they affect purchases of farm tractors. Even though the price of labor may rise less than the price of tractors, the greater increase in production per hour of labor still makes it profitable to exchange labor for tractors. The substitution of tractors for horses began because the greater production possible per man more than offset the higher prices paid for tractors. Thus, if real prices of labor and horses or ratios of these prices to tractor prices were included in a study of demand for farm tractor power, it is likely that their influence would be negative. That is, tractor horsepower increased despite unfavorable price relationships because of the greater financial returns from increased production.

Age of Tractors

The average age of tractors in any given year was derived by accumulating the product of the number of tractors purchased in earlier years that are still on farms by the age of these tractors in the given year, and then dividing this by the total number of tractors still on farms for the year in question and for earlier years. In equation form, this would be:

$$A_{(t)} = \frac{\sum_{i=a}^{t} N_i Y_i}{\sum_{i=a}^{t} N_i}$$

where

$A_{(t)}$ = average age in year t.

N_i = number of tractors purchased in ith year that are still on farms in year t.

Y_i = age of tractors in year t that were purchased in ith year.

a = the oldest year for which tractors still on farms were purchased.

Number of Farms

The numbers of farms are those published by USDA (28). For earlier years they are interpolations between census benchmarks, and for recent years they are projections from the last agricultural census year, 1959. These projections are based largely on data obtained from rural carrier surveys. After the results of the 1964 Census of Agriculture are available, the farm numbers between 1959 and 1964 will be revised if necessary. Annual projections of farm numbers will again be made until information from the next census is available for further evaluation.

The number of farms in this series reflects the change in definition of a farm in the 1959 Census of Agriculture. Revisions were made back to 1950. These revisions had the effect of showing an accelerated reduction in the number of farms between 1950 and 1960.

APPENDIX II: NOTES

Units of Observation for a Durable Good

In the case of a durable good of the type which includes farm tractors, it may well be that periods of observation longer than 1 year might reveal additional information. The entrepreneur's purchases of durable goods might well be based on decisions made over a period of time which is characteristically longer than 1 year. If decision-making time periods are longer than 1 year and are exact intervals of 2 or 3 years, the researcher must select from one of two possible 2-year combinations or one of three possible 3-year combinations. The combination having the highest R^2 or the lowest standard error of estimate might be chosen, but some of the differences result from including and excluding different beginning and ending years. Larger or smaller regression coefficients may add to or subtract from the explanation of changes in the dependent variable associated with changes in the independent variable. Whether the coefficients become larger or smaller depends much upon the positions of the extreme values.

For this problem, longer time periods for each observation will generally yield larger regression coefficients. For a given period of study, use of increasingly longer time periods for each observation reduces the number of observations. As the number of observations approaches the number of variables, the standard error of the regression coefficients tends to get smaller. When the number of observations is equal to the number of variables, the standard error will be zero. Reducing the problem to these proportions is meaningless and does not affect the explained variation of the dependent variables unless there has been a change in the regression coefficients. Thus, any time the number of observations is decreased so as to have observations with longer time periods, a reduction in the standard error of the regression coefficients does not make them more meaningful. But changes in the regression coefficients with subsequent higher R^2 and lower ratios of the standard error of estimate to the mean values might well mean that longer time periods could explain more of the variation in the dependent variable associated with changes in the independent variable.

Interpretation of Additive Variables in Regression Analysis

When the independent variables are additive, the dependent variable is expressed as a function of the sum of all the independent variables. In terms of this problem, each independent variable can change while all others are held constant. Moreover, either the actual or the percentage change in the dependent variable, due to a corresponding change in any one of the independent variables, will be the same regardless of the level of the other independent variables. Here this assumes that tractor horsepower on farms could change by a constant quantity or percentage while all other independent variables are held constant. This assumes also that the real change or percentage change in horsepower purchased associated with changes in horsepower already on farms will not be affected by the level of crop production, price of tractors relative to prices received, size of tractors purchased, age of tractors, and number of farms. Both additive and joint relationships are examined here and it is fairly obvious that the independent variables might be additive either in natural numbers or in logarithmic values. The same line of reasoning will make it nearly as obvious for the other variables.

Multicollinearity and This Analysis

If there is a high degree of multicollinearity in the data, it is difficult to obtain meaningful results with this model. The absence of multicollinearity implies that the independent variables are not highly correlated with each other. Two approaches were explored to compensate for high intercorrelation between the independent variables: First, after determining which variables are closely related to each other, to include only one of the closely related variables in any one problem; second, to get first differences of the original data and then calculate the relationships, that is, the regression coefficients based on first differences.

The first procedure, eliminating all but one of the intercorrelated independent variables, was not considered here because with their elimination the remaining variables would no longer describe purchases of tractors realistically. It is hoped that the advantage of having these coefficients compensates for any lack in preciseness associated with them. The second procedure, getting first differences, was not used because one of the purposes of this study was to estimate the level of actual purchases along with the factors that affected these purchases and the influence of each of the different factors. In using first differences, one would only get the the year-to-year changes in purchases and the related factors, with the influence of each factor related to the changes rather than the level of purchases. In any event, the intercorrelation is not too serious when making projections if it is expected to continue into the future (17).

APPENDIX III: OTHER PROCEDURES CONSIDERED

Several procedures considered for describing past and future changes in the number of tractors included Markov processes and a recursive linear programming technique. The Markov processes were only applied to tractor numbers without considering a system of rewards. The linear programming technique considered costs and described changes in costs, along with the effects of these changes on tractor purchases over time.

Markov Processes

Projections of tractors on farms were made by a particular application of a Markov process. These projections are shown as an alternate procedure to the regression model for estimating changes in tractor purchases. In working with Markov processes, structural relations are specified along with changes in these relations in some period in the past. The solution provides some clues as to possible changes in the overall structure in the future if the internal changes continue as they have in the past. More specifically, as the problem is formulated here, it shows shifts in the number of farms with different levels of income reporting a specified number of tractors. The association of factors with changes is not a part of this model as in regression analysis, except where the variable to be determined may be enumerated in terms of one or more of the related factors.

Here it is concluded that the assumptions inherent in Markov processes are such that the projections from them may not provide much insight into future changes related to changes in tractor numbers on farms. The results are shown merely for purposes of comparison. This discussion is presented to warn the readers that use of this method imposes severe assumptions. The use of this statistical technique assumes that the probability of occurrence in any given time period depends upon the occurrences in the immediately preceding time period and that this dependence remains the same over time. However, this does not mean that it must necessarily be independent of occurrences in earlier time periods.

The problem as formulated here assumes that the probability of a farmer's having one tractor in the first time period and two tractors in the second time period will remain constant over time. Difficulties in aggregation are inherent. These probabilities might ordinarily remain the same over time for a continuing trend in technological change when completely homogeneous groups are specified. When nonhomogeneous farms, such as farms with different incomes, are combined in one group, the average probability for the group is made up of a wide variation of individual probabilities that vary with income. For example, farmers with the higher incomes might have been buying their second tractor during the base period. This would show a high degree of movement from one-tractor to two-tractor farms. Most of the higher income farmers might have purchased their second tractor in the base period, and the probability that the remaining lower income farmers would purchase their second tractor might be considerably lower. It should be clear that this procedure becomes more realistic and useful when estimates are made for homogeneous groups and then aggregated to get a total.

The particular kind of Markov process used here is known as the absorbing Markov chain. Many excellent discussions of the mechanics and manipulation of this statistical tool are available elsewhere (3, 11, 15, (19). Earlier, studies using the Markov chain for analyzing market structure were limited to changes in internal structure (1, 18). But recently some research workers have attempted to use this procedure for estimating changes in the total structure resulting from internal changes. These studies were limited to areas in which there was a decreasing number of farms. However, in this analysis the increasing number of farms necessitates an additional assumption of the highest number of entrants available. Different results are obtained, depending upon the number of entrants available.

Organization of problem

Roughly, the Markov process requires the construction of a matrix of probabilities (transition matrix) for some base period in the past. This matrix is multiplied by itself any number of times-- say "n" times-- to get a new matrix of probabilities in time period(n). These new probabilities indicate the likelihood of being in a specified group after (n) time periods. In this problem, for the farmers who had reported a specified number of tractors in the base period, these new probabilities indicate the likelihood of having a specified number of the tractors after (n) time periods. The result of multiplying these individual probabilities in time period (n) by the number of farms originally having this specified number of tractors is the number of farms now reporting this number of tractors.

The transition matrix of probabilities for this problem was constructed from the number of farms in four different economic classes reporting a specified number of tractors in 1954 and in 1959(table 6). The number of farms reporting a specified number of tractors included those reporting either 0, 1, 2, 3, 4, or 5 or more tractors. These 6 groups for each of 4 economic classes in 1954 and 1959 provide the information needed for constructing a transition matrix of 25 rows and 25 columns. The coefficients in the last row and first column account for farms going out of business or coming into being during the base period. The 2d to 7th rows and columns include the number of farms in the lowest economic class reporting a specified number of tractors. The 8th to 13th rows and columns have similar information for the farms in the 2d lowest economic classes. The 14th to 19th, and the 20th to 25th rows and columns contain the information for farmers in the next higher economic classes, respectively.

The transition matrix used here was constructed by beginning with farmers in the highest economic class. The number of farmers having 5 or more tractors in 1954 was subtracted from the number of farmers having 5 or more tractors in 1959. The additional farmers having 5 or more tractors in 1959 were assumed to come from the same economic class as those who reported 4 tractors in 1954. It was assumed that the remaining farms in 1954 had 4 tractors and that none were added to these farms in 1959. The additional farmers having 4 tractors in 1959 were assumed to come from those having 3 tractors in 1954. Except for the movement of farms from one economic class to another, this procedure is followed to get all elements in the transition matrix.

Table 6.--Number of farms reporting a specified number of wheel tractors in the United States, by economic class of farm, 1954 and 1959

Economic class of farm 1/	Farms reporting the specified number of tractors 2/					
	0	1	2	3	4	5+
Less than $2,500:						
1954	1,693,469	869,377	101,612	12,290	2,470	1,961
1959	855,320	655,668	109,042	15,857	3,279	1,744
$2,500 to $4,999:						
1954	194,824	429,822	159,975	22,336	3,685	1,323
1959	136,644	295,329	148,405	30,147	5,343	1,809
$5,000 to $9,999:						
1954	75,410	287,408	277,131	55,092	9,005	2,883
1959	72,024	219,414	263,491	78,877	15,458	4,617
$10,000 plus:						
1954	46,244	111,828	235,664	117,476	39,570	32,166
1959	57,197	123,629	289,031	196,482	73,349	55,817

1/ Value of farm products sold as estimated by the U.S. Bureau of the Census.
2/ Agricultural census estimates.

In several instances, there was not a sufficient number of farms reporting a given number of tractors available from the farms in 1954 in the same economic class to fulfill the requirements for 1959. This was because of the general movement of farms to higher economic classes. In these instances, it was assumed that farmers with the same number of tractors moved up from the lower economic class to the next higher class.

The numbers in the transition matrix (number of farms with a specified number of tractors in 1954 and 1959) are then converted to probabilities so that the sum of the probabilities in each row adds to unity. This matrix is then squared, cubed, etc., to whatever time period is desired.

The square of the matrix in this case gives probabilities of individual farmers having a specified number of tractors in the first new time period (t+1) after the base period, provided that they had a specified number in the initial year of the base period. In this problem, for example, the probability element in the 25th row and 25th column of the transition matrix squared would be the probability that the number of farms with 5 or more tractors in 1954 still had 5 or more tractors in 1964. The element in the 24th row and 25th column would be the probability of the number of farms that had 4 tractors in 1954 but 5 or more tractors in 1964. Each element in the matrix may be described in a similar fashion.

Similarly, as the transition matrix squared gives probabilities of the farms having a given number of tractors in one time period after having had the same or a different number in some preceding time period, the transition matrix cubed gives results two time periods after the base period. Here, the transition matrix squared refers to estimates for 1964 and the matrix cubed refers to 1969.

When multiplied by the original number of farms in the base period, these probabilities will give the number of farms reporting a specified number of tractors in some future time period. These numbers of farms may be summed for all farms reporting 0, 1, 2, 3, 4, and 5 or more tractors. Then the total number of farms reporting a specified number of tractors can be multiplied by the respective number of tractors to get an estimate of the total numbers. It was assumed that farms reporting 5 or more tractors in future time periods would have about 6.6 tractors per farm, the same as they had during the base period.

Projected tractor numbers on farms

The results of using this procedure with a 1954-59 base indicate that there would be nearly 6 million tractors on farms by late 1969 or early 1970 and almost 7.5 million by late 1979 or early 1980. By using the longer base 1949-59, the increase in tractor numbers would be less rapid, but similar increases would be predicted for later years. Also, by making different assumptions about movements of farmers from one economic class to another, it is possible to show both faster and slower increases in tractor numbers on farms than those shown above. An alternative assumption that projects slower increases in tractor numbers is to assume that farmers will continue to move from one economic class of farm to another in later years as they did during the base period and that the farms moving to a higher economic class of farm will have tractors in the same proportions as those that remained. With this assumption, the number of tractors on each farm in the lower economic group did not affect the chances of an individual farm moving to a higher economic class. Higher results were obtained by assuming that farmers with more tractors were more likely to move to higher economic classes than those with fewer tractors. Different results were obtained by assuming that farmers in all economic classes went out of business. It was assumed that the largest number of farmers who went out of business were from the lowest economic classes and that progressively fewer farmers discontinued operations in the higher economic classes.

Lower results (about 5.6 million tractors in 1969) were obtained by solving the same problem without regard to economic class of farms. By using regional data and disregarding economic classes of farms, about 5.5 million tractors would be estimated to be on farms in 1969. In all of these procedures with the data for tractor numbers and the way the problem has been formulated, the number of tractors would always increase until an equilibrium position had been reached.

Tractor numbers on farms would become stable at the equilibrium position. This does not mean that farmers would not be moving from one group to another. It means that movements in and out of different groups would tend to offset each other so that there would not be a net change. The equilibrium position has not been calculated here because little can be gained from its calculation.

Similarly, matrixes can be estimated showing the mean time for which any one farmer would be in any one size group. These mean times are not calculated here because they too offer little in estimating overall changes in tractor numbers.

For purposes of projections, the internal structure of the problem is important only as it provides good information for the overall objective of getting an aggregate estimate. Thus, mean times in any specified size groups or the equilibrium positions are important in that they are a result of the overall structure of the problem, both internal and external. Aggregate estimates can be no better than the compilation of the results of working with many estimates of the internal structure. However, errors of observation of the internal structure will tend to offset each other, so that an interpretation of the aggregate estimates may be more reliable than the individual elements in the internal structure.

In using Markov processes for making projections, the research worker must use his judgment in deciding how useful or reliable the information may be. For example, this procedure might be considered useful and quite reliable in projecting changes in farm numbers, farm size, and economic classification of farms. Here it might be realistic to assume that farm numbers will tend to decrease and reach a plateau. Likewise, farm size and the proportion of farms in the higher economic classes might be expected to continue to rise much as they have in the past. This statistical technique can be very useful and is relatively easy to use for making such projections.

Even when using the Markov processes for making such simple projections, it may well be merely an exercise in mathematics to calculate the equilibrium state or the mean time of remaining in any size group. One need only look at past changes in, say, size of farms or farm numbers and he will soon realize that these factors will not stabilize in the long run because conditions will not remain as they were during the base period.

It is much more illusory to use this statistical tool for making long-term projections of the demand for a durable good which is subject to the influence of technological innovations. Again, we need only look at history to see that all new machines have been replaced by some other machine before their growth rate had evolved to completion. In the case of tractors, the large 100-horsepower tractors are far different from their 30-horsepower power counterparts of 10 years ago.

The statistical technique can serve as a guide to project short-term changes during certain intervals after some new technology or a new machine has been placed on the market. It might be particularly useful in describing a growth curve in the middle stages if the problem is formulated properly. It could very well be used in describing the adoption rates of a new technology such as hybrid seed corn. It could also provide much information about the rate of adoption after the innovation has been accepted by some individuals and groups. With information related to acceptance by certain groups and their relation to other groups in the population, it would be relatively easy to make a good projection of the adoption of the innovation. This, in turn, would really be an estimate of the demand for the innovation.

Derived tractor purchases

The number of tractors purchased may be derived from the number of tractors estimated to be on farms. Purchases must be large enough to offset discards and to add enough tractors to the stock of tractors on farms so that tractor numbers on farms will increase from what they were in the base period to what they are estimated to be at some time in the future. Using the estimates obtained earlier, tractor purchases would have to be large enough to offset discards between 1959 and 1969 and to add a million tractors to the stock of tractors now on farms.

About 3½ million tractors would have to be bought by farmers between now and 1969, if the projection of tractor numbers on farms in 1969 is correct and if the discard rate continues in the same manner as it did in the late 1950's. About 350,000 tractors, or twice the rate of current purchases, would have to be made annually between 1959 and 1969 to fulfill this estimate. By 1969 then, there would be about 5.7 million tractors on farms as compared with the earlier estimates of between 3.5 and 3.7 million derived from the regression model.

These projections of tractor numbers on farms in late 1979 or early 1980 are obviously too high. The results must be considered in terms of the assumptions of this procedure. This process assumes that the probability of occurrence in any given time depends upon the occurrence in the immediate preceding time period and that this dependence remains the same over time. The usefulness of the Markov process, as applied here, to estimate tractor numbers on farms in the future is questionable. It indicates continuing increases in tractor numbers on farms at a time when they have leveled off and are now declining.

Recursive Linear Programming

The description of the model considered here deals with recursive linear programming. Technical presentations are available elsewhere (7, 24). In general, for this problem recursive linear programming is a mathematical technique which can deal explicitly with the technical and cost aspects of tractor operations in all phases of production, harvesting, and marketing. After technical coefficients have been developed, they are related to costs, income, and noneconomic variables by appropriate research techniques.

The recursive programming technique seems well fitted for forecasting within an economic framework. To use this technique most effectively, the whole organization of the farm should generally be used. This would then measure inputs of all producer goods separately and in total, and allow for any interaction between inputs.

For purposes of this discussion, recursive programming is the general linear programming problem which is static, but made dynamic in a special way. This may be more easily understood if we begin with the general linear programming problem and then relate it to recursive programming. The general problem consists of maximizing or minimizing some goal subject to a set of constraints. In farm management, the goal is generally to maximize farm income with limited amounts of land, labor, and capital. This type of model is static by nature, and the optimum farm program will mean a combination of resources that will give the most income. If such a procedure were used over time, the solutions to the problem would be very erratic. In other words, at the end of 1 year a farm might have 30 dairy cows and 10 hogs. The next year, the optimum program might be 50 hogs and 5 cows, or maybe no hogs or cows at all. If one is interested in income for a given period or a series of years, a procedure that considers the dynamics involved must be used.

One dynamic procedure is to think of the period as a whole. The problem then is to combine the resources in such a way that income for the period will be at a maximum. This type of problem is known as dynamic programming and has the same kind of income equation as the static model, except that the income or profit coefficients are for the whole period rather than for an individual year. Since the problem considers the whole time period, the constraints may be formulated as functions of the resources in the preceding year or years. Thus, what it involves is solving a problem for a given time period in separate stages within the problem. The resources available at each stage are dependent upon what has gone before. For example, if a certain proportion of one's income for a particular year were to be used as capital for the following year, then with positive incomes the restriction on capital would continue to increase in succeeding years throughout the period studied.

Increases in size of farm or in amount of labor could be handled by the same procedure. This type of problem is dynamic in that it includes more than 1 year in the period studied. It states how income could be maximized for the whole period studied, with incomes in succeeding periods related to preceding as well as succeeding years. This two-way causal relation allows for adjustments which might be profitable in later years, but would not be adopted if budgets were made for individual years. This type of problem is subject to the old criticism that most farmers do not achieve maximum incomes, and that for any given period we must begin from the present farm organization and make adjustments within reasonable limits in the direction that would be most profitable.

The recursive programming models presented here are dynamic in that they progress from year to year and begin from the present farm organization. Adjustments over time are made in the direction of maximizing incomes, but are regulated according to certain functions determined by the research worker. These functions might be estimated by regression analysis. Thus, one might say that purchases of tractors depend upon tractor prices, prices of other inputs, and prices of farm products. Solutions from these simple or multiple relationships may be considered in a recursive programming problem by using sets of equations that would allow solutions within a specified range above and below the relationships determined by the regression analysis.

In working a recursive programming problem, one can use all of the knowledge gained from the average relationships determined by correlation and regression analysis. In addition, one considers the forces related to profit motivation in arriving at annual estimates of the unknown parameters--here, numbers of tractors on hand and tractor purchases.

The possible use of recursive programming procedures for studying the demand for farm tractors was explored by several different approaches. One exploration is shown below. This exploration was rejected in the final analysis because of the lack of data. An elaboration of this point follows.

Specification of problem

The problem outlined here includes costs, quantity of tractors on hand, and purchases for farms of different size groups, each group having several sizes of tractors. The tractors in turn are further grouped by age. Thus, the smallest farms as a group may have several sizes of tractors. Both the small and large sizes of tractors on small farms may be new, middle-aged, or old. The same classification holds for the larger sizes of farms. If farms were divided into three size groups, each having three sizes of tractors in three separate age classes, there would be $(3)^3$, or 27, different groups to consider. Each of these groups will be represented by an average organization. These individual farms are then aggregated to get area totals.

Whereas in many farm management studies the objective or profit function is maximized, here a cost function for tractor operations will be minimized to reduce the size of the problem. It is assumed that in order for this farm to operate it must perform all the usual preharvesting and harvesting functions for all crops. Therefore, if a farmer could estimate the lowest cost of harvesting these crops, he would at the same time be making adjustments so that his income from this input could be maximized for the existing crop pattern. This is consistent with the marginal productivity theory of the firm which states that the firm should operate along the "expansion path" or line of least cost for producing each level of output. The crop pattern is taken as given, and here it is assumed that the changes are independent of the number of tractors.

The cost coefficients will change each year and will represent the total cost of performing all the operations on all farms of a particular group with the given size and age of tractor, if all the work were done with that source of power. The coefficient for each size group of farms is the cost per farm multiplied by the number of farms in the size group. Between years, these costs will vary for a given size and age of tractor because of changes in the purchase price, operating costs, efficiency, technology, types of operations performed, and number of farms. Changes in the number of farms for each size group will be made as shown by census data, and projected as required.

The unknown in the cost equation is the proportion of work that will be done by each size and age of tractor. The sum of these unknowns for each size of farm will have to add up to 1. Therefore, on the smaller farms, the first unknown is the proportion of the work that will be done on all of the smallest farms by new tractors of the smallest size. The next unknown is the proportion of the work that will be done on all of the smaller farms by the smallest middle-aged tractors, and so on. The value of the cost equation as such is not important with this formulation of the problem. It is only the relative values that will influence the purchase of tractors.

It remains to be shown how the cost equation is related to purchases and numbers of tractors. As has been stated, the cost of harvesting all crops on individual farms by a particular procedure must be estimated. Then, numbers of tractors may be obtained by dividing this total cost by an average cost normally attributed to a particular size of tractor. This cost per tractor will be the product of the cost per hour and the number of hours. The cost per hour will be based on variable as well as fixed costs and will vary by the size and age of tractor. The number of hours will be estimated from base period figures as determined by surveys in the area. The number of tractors so estimated should approximate known data for any given year.

Thus far, for each size group of farms and size-and-age group of tractors we have estimated the number of tractors which are required to harvest these crops for some base year. The decision must still be made as to replacements and additions over time. From one year to the next, the number of tractors on hand in the first year, plus the number purchased, minus those being scrapped or moving into another age group, will equal the number on hand the following year. This relationship was built into the model in this form by stating it as an equality. In other words, this condition must hold.

The numbers of used tractors in different age groups are functions of the purchases of these tractors in earlier periods. That is, the number of 25-horsepower tractors that might be considered to be in the medium age group would be the number of tractors purchased 4 to 8 years previously, less those scrapped. Likewise, the number of old tractors would be the number of tractors purchased 12 to 15 years earlier, minus those scrapped. This way, depreciation may be built into the model as a discrete frequency distribution of the step-function type rather than a single figure.

The number, age, and size of tractors on farms as estimated in this problem are such that the cost of doing all the operations required to harvest the crops normally grown on these farms is the least possible within certain limitations. Changes in optimum tractor numbers from year to year are due to changes in price relationships, changes in technology, and changes in age and size distribution. This optimum number of tractors is usually not realized in any given year because farmers are not willing or able to make the changes. Changes in numbers from one year to the next are restricted in this problem by a set of equations on the basis of historical changes resulting from similar forces. In the past these forces encouraged shifts in the number of tractors per farm related to crop production per man-hour. This is considered in the problem by including another set of equations limiting changes in number of tractors within a specified range above and below the number of tractors expressed as a function of crop production per man-hour in the previous year. Thus far, within these relationships, number of tractors of a given age and size for any year will be the number with which the crop acreage can be harvested at the lowest possible cost.

The actual number of tractors on hand in any year will depend upon beginning inventories, purchases, and scrapping. Over time, purchases must be great enough to replace old and worn-out tractors and to add more new tractors to take advantage of less costly and easier ways of harvesting crops. The level of purchases for any input depends not only upon the state of innovation and available capital but also upon changes in technology.

For any given year, purchases may fluctuate because of changes in prices of tractors, products, other factors, and so on. All these factors affect gross income. These ideas are considered here by assuming that net purchases are expressed as a function that will vary directly with income in the preceding year or two and inversely with the number of tractors on hand, and that replacement will be equal to depreciation unless net purchases are negative. This concept is included in the problem by adding a set of equations that place upper and lower limits on the purchases of tractors. Within these limits, annual purchases will be the additions needed to harvest the crop acreage at the lowest cost if, and only if, this is consistent with the aforementioned historical relationship and the number of tractors expressed as a function of crop production per man-hour the previous year.

Complexity of recursive programming approach to this problem

This procedure for investigating the demand for farm tractors was not used because the data requirements were enormous. To determine the demand for farm tractors in the United States, some kinds of typical farms would have to be constructed for all of the type-of-farming areas. The number of farms required for each area would depend on the number of farms having similar proportions of resources and costs of production. A detailed discussion of the requirements for proportionality in the resource requirements and in the objective functioning of linear programming problems appears in the Journal of Farm Economics (8). Little is known of the effects of using farm organizations that deviate from constant proportions.

For each type and size of typical farm, the cropping pattern would need to be determined. Adjustments in the cropping pattern over time would be made toward those crops returning the largest profits. Errors in the estimates of cropping patterns would not change the solution if they affected only the levels of costs and relative costs for harvesting crops.

Data requirements

The data required include inputs such as fertilizer, seed, spray materials, etc. They also include estimates of production for the various input levels or categories. Prices are needed for the inputs for all the factors of production of individual inputs and also of all goods and products sold. These data are required by regional areas or for typical organizations within an area. The solutions one gets to these problems depend upon the input levels of the various inputs. Errors in these estimates are generally ignored in solutions of problems to date. Currently, most problems have been used for the analysis of supply response. This means they have been used to estimate changes in livestock numbers and crops. Many of these broad general solutions have concentrated on the area approach rather than the individual farm approach. However, it is argued that flexibility constraints be included to reflect adjustments for individual farms in a specific area. In setting up these programming models, different levels of technology are used so that the solution will have the most profitable level of technology. This level will change over time, depending upon changes in input and product prices. It has been shown that small changes in the level of technology have not affected the solutions.

Since it has been shown that small changes in technology will not affect the recursive programming solutions, it is doubtful that such a problem can be sensitive enough to estimate changes in the use of various technologies on farms. This is especially true since many farmers buy tractors and farm machines for reasons other than to maximize their profits.

To study the demand for farm tractors and other farm machinery, or for that matter other durable inputs, would require models of typical farms or area models of the United States for many different farms under various operating conditions. Now that the direction of agricultural research is such that these kinds of farm organizations are being constructed, they might prove to be useful for inserting various technological changes in farm practices so that one could see the effect on income. Or by including these activities showing different rates of technology, one could also see the different adjustments and rates of adjustments of cropping patterns and income over time. When these models are available, this study may be further explored to determine its usefulness in answering the question being raised here.

One way of considering the alternative technologies and the direction and rate of adoption would be to include all possible alternatives such as plowing, disking, harrowing, planting, and cultivating for each of several crops. This would result in a selection of the machines over time which would contribute to higher incomes. This procedure, if applied to only one farm practice, would result in an enormous problem. The application to several practices would be unthinkable.

Another procedure would involve the selection of the least-cost combination of machines which could be used for crop production in the last several years. With the inclusion of flexibility constraints in the problem and by obtaining a series of solutions over time, an adjustment path would show changes in types, sizes, and numbers of machines. This problem might be worked independently from the rest of the farm plan, but the results could be included in the construction of flexibility constraints for the overall model. With a preliminary solution of the overall model, the above procedure could again be used for obtaining new estimates of machinery based on the new crop rotation.

LITERATURE CITED

(1) Adelman, I. G.
1958. A Stochastic Analysis of the Size Distribution of Firms. Jour. Amer. Statis. Assoc., 53:893-904, Dec.

(2) Allen, R. G. D.
1956. Mathematical Analysis for Economists. MacMillan & Co., Ltd., New York, N.Y.

(3) Bostwick, Don
1962. Yield Probabilities as a Markov Process. Agr. Econ. Res. 14(2):49-56, Apr.

(4) Brodell, Albert P., and Kendall, Albert R.
1950. Life of Farm Tractors. U. S. Dept. Agr., FM-80. June.

(5) Cromarty, William A.
1960. The Demand for Farm Machinery and Tractors. Mich. State Univ., Tech. Bul. 275, East Lansing, Mich.

(6) Csorba, J. J.
1959. The Use of Horses and Mules on Farms. U. S. Dept. Agr., Agr. Res. Serv., ARS 43-94, Mar.

(7) Day, Richard
1963. Recursive Programming and Production Response. North-Holland Pub. Co., Amsterdam, The Netherlands.

(8) ________
1963. On Aggregating Linear Programming Models of Production. Jour. Farm Econ. 45(4):797-813, Nov.

(9) Ezekiel, Mordecai, and Fox, Karl A.
1959. Methods of Correlation and Regression Analysis. John Wiley & Sons, Inc., New York, N.Y.

(10) Farm Equipment Institute
1964. United States-Made and Imported Wheel Tractors by End Use, January-December 1962 and 1963. FEI Retail Sales Rpt. 13867, Chicago, Ill., Jan. 30.

(11) Feller, William
1960. An Introduction to Probability Theory and Its Application. John Wiley & Sons, Inc., New York, N.Y.

(12) Foote, Richard J.
1958. Analytical Tools for Studying Demand and Price Structures. U. S. Dept. Agr., Agr. Mktg. Serv., Agr. Handb. 146, Aug.

(13) Griliches, Ziv
1960. The Demand for a Durable Input: Farm Tractors in the United States, 1921-57. In Harberg, Arnold C.(Editor), The Demand for Durable Goods. Univ. Chicago Press.

(14) Heady, Earl O., and Tweeten, Luther G.
1963. Resource Demand and Structure of the Agricultural Industry. Iowa State Univ. Press, Ames.

(15) Howard, R. A.
1960. Dynamic Programming and Markov Processes. Technology Press, Cambridge, Mass., and John Wiley & Sons, Inc.

(16) Hurst, W. M., and Church, L. M.
1933. Power and Machinery in Agriculture. U. S. Dept. Agr., Bur. Agr. Econ., Misc. Pub. 157.

(17) Johnston, J.
1963. Econometric Methods. McGraw-Hill Book Co., Inc., New York, N. Y.

(18) Judge, G. G., and Swanson, E. R.
1961. Markov Chains: Basic Concepts and Suggested Use in Agricultural Economics. Univ. Ill., Dept. Agr. Econ., Agr. Expt. Sta., AERR-49, Urbana, Ill.

(19) Kemeny, J. G., and Snell, J. L.
1960. Finite Markov Chains. Van Nostrand Co., Inc., Princeton, N. J.

(20) Matthews, R. C. O.
1960. The Business Cycle. Univ. Chicago Press.

(21) Mood, Alexander M., and Gaybill, Franklin A.
1963. Introduction to the Theory of Statistics. McGraw-Hill Book Co., Inc., New York, N. Y.

(22) Nerlove, Marc
1958. Distributed Lags and Demand Analysis for Agricultural and Other Commodities. U. S. Dept. Agr., Agr. Mktg. Serv., Agr. Handb. 141, June.

(23) Parsons, Merton S., Robinson, Frank H., and Strickler, Paul E.
1960. Farm Machinery: Use, Depreciation, and Replacement. U. S. Dept. Agr., Agr. Res. Serv., Statis. Bul. 269, Oct.

(24) Schaller, W. Neill, and Dean, Gerald W.
1965. Predicting Regional Crop Production--An Application of Recursive Programing. U. S. Dept. Agr., Econ. Res. Serv., Tech. Bul. 1329.

(25) Strickler, Paul E., and Hines, Charles A.
1960. Numbers of Selected Machines and Equipment on Farms. U. S. Dept. Agr., Agr. Res. Serv. and Agr. Mktg. Serv., Statis. Bul. 258, Feb.

(26) Tinbergen, J.
1949. Long-Term Foreign Trade Elasticities. Metroeconomica (1):174-185. Trieste.

(27) U. S. Department of Agriculture
1959. Farmers' Expenditures for Motor Vehicles and Machinery With Related Data, 1955. Agr. Mktg. Serv. and Agr. Res. Serv., in cooperation with U. S. Dept. Commerce, Statis. Bul. 243.

(28) ________ 1962. Number of Farms 1910-1959, Land in Farms 1950-1959, by States. Statis. Rptg. Serv., Statis. Bul. 316, June.

(29) ________ 1963. Changes in Farm Production and Efficiency. A Summary Report. Statis. Bul. 233, July. (Revised.)

(30) ________ 1963. Farm Income Situation. Econ. Res. Serv., FIS-191, July.

(31) U. S. Department of Commerce
1920-62. Current Industrial Reports. Tractors (Except Garden Tractors). Bureau of the Census Series:M35S.

(32) ________ 1920-62. Overseas Business Reports. Bur. of Internatl. Commerce.